ON THE ANTIQUITY OF THE STAR COORDINATES FROM INDIAN JYOTISHA SHASTRAS

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A comparison is made between coordinates for 35 stars listed in traditional Indian astronomical texts and the coordinates of corresponding stars listed in modern tables. We find that the vectors pointing from the Indian star positions to the corresponding modern star positions are strongly correlated with the reversed proper motion vectors of the stars. Once precession is taken into account, the modern star positions show a tendency to move towards the Indian star positions as we go back in time.

We present the hypothesis that the Indian star positions show this correlation because they were measured in the distant past. We attempt to use the proper motion correlation to date the Indian star coordinates, and we find that they divide into a group about 50,000 years old, and a group a few thousand years old. There is also a group that cannot be clearly dated, and this may be partly due to errors in star identification in this group.

In Chapter 8 of the traditional Indian astronomy text called *Surya-siddhanta*, there is a list of coordinates for 35 stars. These stars include the yogataras (literally, "junction stars") of 28 constellations, called nakshatras. The nakshatras are distributed at roughly equal intervals around the ecliptic and serve as markers indicating the motion of the Sun and Moon. In addition, coordinates are listed for 7 individual stars which are not part of the nakshatras.

In other Indian astronomical texts, there are similar lists of star coordinates. In Tables 1a and 1b there are lists of star coordinates from 6 different texts. Three of these texts are the *Paitamahasiddhanta* from the *Visnudharmottara Purana* (column 1), the *Surya-siddhanta* (column 2), and the *Brahmagupta Siddhanta* (column 3). In the opinion of David Pingree (1989, pp. 102–103), the *Paitamahasiddhanta* is the oldest known star catalogue of yogataras of nakshatras.

On encountering this list of Sanskrit star names, it is natural to ask which stars they correspond to in modern astronomy. It turns out that it is not easy to obtain clear identifications for many of these stars. It appears that for many centuries, Indian astronomers have not been concerned with directly observing stars or measuring their positions. Thus, in the 10th century A.D. the Muslim astronomer

TABLE 1A

Polar Longitudes for Stars from Six Indian Astronomical Works

	Sanskrit Star Name	Polar Longitudes (deg.; min.) 1	2	3	4	5	6
1	Asvini	8;	8;	8;	8;	8; 30	8;
2	Bharani	20;	20;	20;	20;	21; 15	21;
3	Krittika	37; 28	37; 30	37; 28	36;	37; 45	38;
4	Rohini	49; 28	49; 30	49; 28	49;	49;	49;
5	Mrigasirsa	63;	63;	63;	62;	62;	62;
6	Ardra	67;	67; 20	67;	70;	66;	66;
7	Punarvasu	93;	93;	93;	92;	92; 45	94;
8	Pushya	106;	106;	106;	105;	106;	106;
9	Aslesha	108;	109;	108;	114;	107; 15	106;
10	Magha	129;	129;	129;	128;	129;	129;
11	Purvaphalguni	147;	144;	147;	139; 20	148;	148;
12	Uttaraphalguni	155;	155;	155;	154;	155; 30	155;
13	Hasta	170;	170;	170;	173;	170;	170;
14	Chitra	183;	180;	183;	184; 20	183;	183;
15	Svati	199;	199;	199;	197;	198; 30	198;
16	Visakha	212; 05	213;	212; 05	212;	212; 15	212;
17	Anuradha	224; 05	224;	224; 05	222;	224; 15	224;
18	Jyestha	229; 05	229;	229; 05	228;	229; 30	230;
19	Mula	240; 04	241;	241;	241;	242;	242;
20	Purvashadha	249;	254;	254;	254;	255; 30	255;
21	Uttarashadha	260;	260;	260;	267; 20	260;	261;
22	Abhijit	265;	266; 40	265;	267;	259; 45	258;
23	Sravana	278;	280;	278;	283; 10	275; 15	275;
24	Dhanishtha	290;	290;	290;	296; 20	287; 30	286;
25	Satabhisaj	320;	320;	320;	313; 20	320;	320;
26	Purvabhadrapada	326;	326;	326;	327;	325;	325;
27	Uttarabhadrapada	337;	337;	337;	335; 20	337;	337;
28	Revati	0;	359; 50	0;	359;	0;	0;
29	Agastya	87;	90;	87;	87;		80;
30	Mrigavyadha		80;	86;	86;		81;
31	Agni		52;				53;
32	Brahmahridaya		52;				55;
33	Prajapati		57;				61;
34	Apamvatsa		180;				183;
35	Apas		180;				

The first set, in column (1), is from the *Paitamahasiddhanta* of the *Visnudharmottara Purana* (Pingree 1989, p. 103). The remaining five columns of star coordinates are listed by S. B. Dikshit (1969, pp. 338–339). These are (2) *Surya-siddhanta*, (3) *Brahmagupta Siddhanta*, (4) *Lalla Tantra*, (5) *Damodariya B. Tulya*, (6) *Graha-laghava*.

TABLE 1B

Polar Latitudes for Stars from Six Indian Astronomical Works

	Sanskrit Star Name	Polar Latitudes (deg.; min.) 1	2	3	4	5	6
1	Asvini	10;	10;	10;	10;	10;	10;
2	Bharani	12;	12;	12;	12;	12; 15	12;
3	Krittika	5;	5;	4; 31	5;	4; 30	5;
4	Rohini	-5;	-5;	-4; 33	-5;	-4; 30	-5;
5	Mrigasirsa	-10;	-10;	-10;	-10;	-10;	-10;
6	Ardra	-11;	-9;	-11;	-11;	-11;	-11;
7	Punarvasu	6;	6;	6;	6;	6;	6;
8	Pushya	0;	0;	0;	0;	0;	0;
9	Aslesha	-7;	-7;	-7;	-7;	-7;	-7;
10	Magha	0;	0;	0;	0;	0;	0;
11	Purvaphalguni	12;	12;	12;	12;	11; 45	12;
12	Uttaraphalguni	13;	13;	13;	13;	12; 45	13;
13	Hasta	-11;	-11;	-11;	-8;	-11;	-11;
14	Chitra	-2;	-2;	-1; 45	-2;	-1; 45	-2;
15	Svati	38;	37;	37;	37;	37; 15	37;
16	Visakha	-1; 30	-1; 30	-1; 23	-1; 30	-1; 15	-1;
17	Anuradha	-3;	-3;	-1; 44	-3;	-1; 45	-2;
18	Jyestha	-4;	-4;	-3; 30	-4;	-3; 30	-3;
19	Mula	-8; 30	-9;	-8; 30	-8; 30	-8; 30	-8;
20	Purvashadha	-5; 20	-5; 30	-5; 20	-5; 20	-5; 30	-5;
21	Uttarashadha	-5;	-5;	-5;	-5;	-5;	-5;
22	Abhijit	62;	60;	62;	63;	62;	62;
23	Sravana	30;	30;	30;	30;	29; 30	30;
24	Dhanishtha	36;	36;	36;	36;	25; 30	36;
25	Satabhisaj	0;	-0; 30	-0; 18	-0; 20	-0; 15	0;
26	Purvabhadrapada	24;	24;	24;	24;	23; 45	24;
27	Uttarabhadrapada	26;	26;	26;	26;	26;	27;
28	Revati	0;	0;	0;	0;	0;	0;
29	Agastya	-76;	-80;	-77;	-80;		-76;
30	Mrigavyadha		-40;	-40;	-40;		-40;
31	Agni		8;				8;
32	Brahmahridaya		30;				30;
33	Prajapati		38;				39;
34	Apamvatsa		3;				3;
35	Apas		9;				

These correspond to the polar longitudes listed in Table 1a.

Albiruni wrote in his book *Indica* that he was unable to find an Indian astronomer who would point out for him all the stars and constellations of the Indian system. The same thing has been reported in recent times by various European and American students of Indian astronomy.

As a result, it is necessary to arrive at star identifications by a comparative study of three sources of information: (1) testimony from traditional Indian astronomers, (2) descriptions of the stars and constellations in various shastras, and (3) the quantitative comparison of Indian star coordinates with modern star coordinates. When this is done, some identifications seem to be quite clear, and scholars tend to be in unanimous agreement about them. Others remain ambiguous, and there are conflicting opinions.

Once a list of star identifications is drawn up, it is possible to evaluate how close the Indian coordinates are to the modern coordinates of the corresponding stars. The coordinates from the *Surya-siddhanta* and the different works listed in Tables 1a and 1b are polar longitudes and latitudes: For a given star, one can draw half of a great circle from the north celestial pole, through the star, and on to the south celestial pole. This half circle intersects the ecliptic at a certain point. The angular distance from this point to the star is called the polar latitude (and we write it as positive or negative depending on whether the star is to the north or south of the ecliptic). The longitude of this point, measured along the ecliptic from the vernal equinox, is called the polar longitude.

Polar longitudes and latitudes can be converted into the coordinates of right ascension and declination used in modern astronomy (see Appendix). To compare these with modern star coordinates, it is necessary to take into account the precession of the equinoxes. If we assume that the Indian star coordinates were intended for a particular epoch, we can transform them from that epoch to the epoch of A.D. 2000 using the formulas for precession. Then they can be compared with modern coordinates tabulated for this epoch.

When this is done for the best available sets of star identifications, it is found that the average difference between the old Indian and the modern star positions is minimal near the end of the 5th century A.D. This average difference is a bit more than 2 degrees, with some differences as large as 5, 6, or even 7 degrees.

Some scholars interpret this state of affairs as proof that Indian astronomers have never made astronomical measurements, and that their astronomical systems have therefore been imported from other countries, such as Greece, Babylon, or China. For example, David Pingree maintains that the star catalogue in the *Paitamahasiddhanta* "is more likely to be an Indian adaptation of a Greek star catalogue than to be based on observations that were made in India; and that the ineptitude with which the Indians historically tried to 'correct' these coordinates militates against any theory that is founded upon the idea that the Indians of the medieval period were experts in astronomical observation"

(Pingree 1989, p. 99). Pingree argues that the transmission of star coordinates from Hellenistic countries to India probably occurred in the 5th century A.D.

If this interpretation is accepted, then it follows that the Greek astronomers from whom the Indians borrowed were rather poor observers, and customarily made errors of several degrees in measuring star positions. This is odd, since the star catalogue of the Hellenistic Egyptian astronomer, Ptolemy, is much more accurate, and it was compiled in the 2nd century A.D. Pingree's answer to this objection is that the Indians in the 5th century somehow stumbled on a cruder pre-Ptolemaic Greek tradition which was later eradicated by Ptolemaic influence in countries west of the Indus river, but which survived in India.

Another interpretation is that the Indian star coordinates may belong to an indigenous Indian tradition of great antiquity. According to this idea, Indian astronomers did not make observations of stars during the medieval period (or, at least, they did not correct their tables on the basis of such observations). Rather, they simply preserved star catalogues dating from the remote past on the authority of ancient tradition. Such star catalogues may have been mathematically transformed in the late fifth century into a format reflecting that epoch. This step is plausible, and of course Pingree's theory requires that it must have been carried out with an old Greek star catalogue. We will discuss how it could have been done later on in this paper.

According to this interpretation, it is not necessarily true that Indians were unaccustomed to making astronomical observations during the Middle Ages. They may have regularly observed the planets, since planetary parameters of motion are always changing and need to be updated. But since the stars remain almost stationary in their positions on the celestial sphere, they may have felt no need to make new observations of their coordinates. This lack of a need to observe the stars may account for the fact that many Indian astronomers of recent centuries were apparently not acquainted with the locations in the sky of several nakshatra constellations.

Whether this interpretation is correct or not, there does seem to be evidence suggesting that the Indian star coordinates are very old. Interestingly enough, this evidence is based on the fact that the fixed stars do move slowly across the celestial sphere. These slow motions of stars are called proper motions.

Halley first observed the proper motions of stars in 1718, when he ascertained that Arcturus and Sirius had shifted in position by a degree and half a degree, respectively, since the time of Ptolemy (Motz and Duveen 1977, p. 450). Today, Arcturus and Sirius are said to move across the celestial sphere at rates of about 2.3 and 1.3 seconds of arc per year, respectively. In contrast, most stars visible to the naked eye move at rates from a tenth down to a few hundredths of a second per year. Their motion is often measured by comparing their positions on photographic plates taken with powerful telescopes over intervals of several years.

Using the information we have for the motion of Arcturus, we can compute that the time of Ptolemy must have been about 1718-3600/2.3 = A.D. 152.8. This figure agrees well with historical records of Ptolemy's life span, but the figure of 1718-1800/1.3 = A.D. 333.4, obtained from the data for Sirius, is a bit high. This shows that proper motion data can be used to date star catalogues, but that the dates may show a considerable spread due to various errors.

In this paper, we will try to use proper motions of stars to date the old Indian star catalogues. Our method is as follows: First, select a given set of coordinates from Tables 1a and 1b. Since we wish to consider a full set of 35 stars, we fill out the table using *Surya-siddhanta* coordinates in cases where coordinates for some of the stars 29–35 are missing.

The second step is to select a set of identifications of Sanskrit star names with modern star names. Such star identifications are discussed in Pingree (1989), Kay (1981), Dikshit (1969), Burgess (1860), and Colebrooke (1807). Two strategies generally seem to be employed in making star identifications: (1) to seek relatively bright and distinctive stars and constellations as close as possible to the Indian star positions, and (2) to seek dim stars that are much closer to these positions. We tend to favor the first strategy, since there are many dim stars to choose from, and it is hard to see how the choices made by various authors can be justified. We will therefore rely mainly on the identifications made by Burgess, which follow the first strategy.

The third step is to define two vectors for each modern star name that is identified with a Sanskrit star name. Let X be the modern star position at the date A.D. 2000, and let Y be the corresponding Indian star position precessed from its epoch up to A.D. 2000. The error vector is the vector Y-X pointing from the modern star position to the Indian star position. The distance D from X to Y in degrees is the size of the error. (In the Appendix, we discuss the mathematical details involved in these definitions. The actual calculations were performed using 3-dimensional vector analysis. But for simplicity, we can think of the positions X and Y, and the various vectors, as lying in a plane tangent to the celestial sphere at the Indian star position, Y. This enables us to define various angles and distances in terms of plane geometry.)

Burgess (1860, p. 355) computes the average of the errors in longitude of the 28 nakshatra junction stars relative to their corresponding modern stars. This average is 0 at about A.D. 490. Our own calculations show that, as one would expect, the average of the distances D for these 28 stars comes close to its minimum at this date. Therefore, we primarily use A.D. 490 as the epoch for the Indian star coordinates.

The epoch we use for modern star coordinates is A.D. 2000, since the most up-to-date catalogues are computed for this date. When comparing star coordinates corresponding to different dates, we will always want to precess them from their

epoch to the epoch of A.D. 2000, just as we have done for the Indian star coordinates. In particular, we will be interested in X(t), the modern star position 10,000t years before A.D. 2000. This position is assumed to be precessed up to A.D. 2000, and thus it differs from X solely due to the effects of proper motion. If the proper motion is 0 then X(t) is always the same as X, and if it is not 0, then X(t) will slowly retrace the past motion of the star as t varies. We define D(t) to be the distance in degrees from X(t) to Y.

The second vector, V, is the reverse of the proper motion vector of the star, and its coordinates are expressed in units of degrees per 10,000 years. Since proper motion is essentially rectilinear over a few degrees, X(t) is approximately the same as X+Vt. We will use B.E. to mean before the modern epoch of A.D. 2000. For example, in 50,000 B.E. the position of the star due to proper motion would be X(5), and this is about X+5V. Since V is the *reversed* proper motion vector, adding 5V means shifting to the position 50,000 years before A.D. 2000. D(5) is the distance in degrees between the position of Y on the celestial sphere and the position of the star in 50,000 B.E.

Our hypothesis is that the Vedic star coordinates refer to a distant time, which we can call 10,000T B.E. If this hypothesis perfectly accounted for the differences between modern and Indian star coordinates, then we would expect to find Y to be nearly equal to X(T) for each pair of Indian and modern coordinates. In that case, we could solve the equation Y=X+TV for T, and thus arrive at a date for the Indian coordinates.

Unfortunately, things are not that simple. Various errors could effect Y, the Indian star position. These include copying errors, round-off errors (since only a very few fractions of a degree are used in the coordinates in Tables 1a and 1b, pp. 2–3), and star identification errors. Perhaps there are even errors in the modern determinations of proper motions. (We have run into errors in modern star tables.)

If there is an error in Y or V (or X, for that matter), then we would expect X+TV to miss Y as we go back to earlier and earlier dates. Let the angle between V and the error vector Y-X be called A, the angle of approach. A should be 0 if there are no errors in Y, V, or X, but if there are errors in these vectors, then A will be non-zero.

1. STATISTICAL ANALYSIS OF THE ANGLES OF APPROACH

This leads to the following question: Does the distribution of the angles of approach tend to support our hypothesis concerning the antiquity of the Indian star coordinates, or is it more consistent with some other hypothesis? In this section we will approach this question from a statistical point of view, in which

TABLE 2

Indian Star Coordinates Compared with Modern Coordinates

No.	Sanskrit Name	Modern Name	D	D(5)	A	Magn.
19	Mula	Lambda Scorpionis	5.016	4.614	3.826	1.63
32	Brahmahridaya	Alpha Aurigae	6.487	.531	4.011	.08
29	Agastya	Alpha Carinae	.846	.377	7.447	-.72
4	Rohini	Alpha Tauri	1.650	1.579	8.524	.85
15	Svati	Alpha Bootis	2.851	25.574	8.638	-.04
3	Krittika	Eta Tauri	.613	.114	8.750	2.87
13	Hasta	Delta Corvi	2.839	.942	9.895	2.95
33	Prajapati	Delta Aurigae	6.571	4.687	22.397	3.72
7	Punarvasu	Beta Geminorum	.689	8.189	26.553	1.14
1	Asvini	Beta Arietis	1.271	1.043	27.117	2.64
23	Sravana	Alpha Aquilae	.709	6.660	32.233	.77
35	Apas	Delta Virginis	5.910	3.615	33.833	3.38
22	Abhijit	Alpha Lyrae	1.680	3.230	36.023	.03
14	Chitra	Alpha Virginis	.943	.582	38.093	.98
34	Apamvatsa	Theta Virginis	1.844	1.374	39.022	4.38
25	Satabhisaj	Lambda Aquarii	.673	.435	40.104	3.74
5	Mrigasirsa	Lambda Orionis	4.397	4.342	40.757	3.39
10	Magha	Alpha Leonis	.214	3.336	43.313	1.35
18	Jyestha	Alpha Scorpionis	1.499	1.326	53.067	.96
2	Bharani	35 Arietis	1.289	1.215	57.131	4.66
16	Visakha	24 Iota Librae	2.586	2.290	59.600	4.54
30	Mrigavyadha	Alpha Canis Major	1.119	15.133	62.610	-1.46
31	Agni	Beta Tauri	7.925	7.189	63.972	1.65
8	Pushya	Delta Cancri	1.751	3.006	64.763	3.94
24	Dhanishtha	Beta Delphini	3.203	2.979	67.581	3.54
26	Purvabhadrapada	Alpha Pegasi	3.459	3.226	68.341	2.49
17	Anuradha	Delta Scorpionis	3.053	3.026	82.294	2.32
9	Aslesha	Epsilon Hydrae	5.230	5.774	85.337	3.38
6	Ardra	Alpha Orionis	6.261	6.251	86.837	.50
27	Uttarabhadrapada	Alpha Andromedo	5.779	6.524	92.205	-.16
20	Purvashadha	Delta Sagittarii	1.251	1.604	111.208	2.70
28	Revati	Zeta Piscium	1.182	2.989	125.959	4.86
12	Uttaraphalguni	Beta Leonis	.411	7.415	145.010	2.14
21	Uttarashadha	Sigma Sagittarii	2.306	2.976	146.561	2.02
11	Purvaphalguni	Delta Leonis	3.768	6.136	151.184	2.56

Indian coordinates for stars 1–30 are from the *Brahmagupta Siddhanta*, and those for stars 31–35 are from *Surya-siddhanta*. Modern star identifications for stars 1–28 are from Burgess (1860, p. 355), and those for stars 29–35 are from Burgess (1860, p. 364).

The tabulated quantities are: D, the distance from the modern star position on the celestial sphere to the Indian star position; D(5), the distance from what the modern star position would have been in 50,000 B.B. to the Indian star position; and A, the angle of approach (see the text). Magn is the visual magnitude of the star.

The table entries are listed in order of increasing angle of approach. The average of these angles is 55.79 degrees.

we look at the set of A's as a whole. In the next section, we will adopt the alternative approach of examining the 35 sets of star coordinates individually in an effort to see whether or not they fit our hypothesis.

Since proper motions of stars are very small, it would be natural to suppose that no one knew about them before the development of modern astronomical methods. Thus it would be natural to suppose that there should be no relationship between proper motions of stars and errors in the rather inaccurate looking Indian star coordinates. This is one obvious alternative to our hypothesis that these coordinates are old enough to reflect the effects of stellar motions. If this alternative view is correct, then one would expect the angles of approach to be scattered more or less uniformly between their minimum value of 0 and their maximum value of 180 degrees.

In Table 2, there is a listing of values of D, D(5), A, and visual magnitudes for a particular set of star coordinates and identifications. The Indian coordinates for stars 1–30 are from the *Brahmagupta Siddhanta*, and those for stars 31–35 are from *Surya-siddhanta*. We have made this choice of coordinates for the following reasons: For stars 31–35 the *Surya-siddhanta* coordinates are the only ones we have, and so we have no choice but to use them. For the remaining stars, the coordinates from the *Brahmagupta Siddhanta* seem to be more accurate than those of the *Surya-siddhanta*, and thus we have used them for most of our calculations.

We take modern star identifications from Burgess (1860). The identifications for the 28 nakshatras include the yogataras, which are listed in Table 2, as well as the additional stars making up these constellations. The identifications of the yogataras and stars 29–35 are tabulated in Burgess (1860, pp. 355, 364). The constellations themselves, along with stars 29–35, are mapped on the celestial sphere in Figure 1 (pp. 10–11), using modern star coordinates for A.D. 2000. Our justification for using this set of identifications is that Burgess clearly gives the reasoning behind his choices, and they are based mainly on prominent stars rather than on stars of low magnitude.

The table entries are listed in order of increasing angle of approach. We can see that there are 29 with A less than 90 degrees and only 6 with A greater than 90 degrees. Given the assumption that the A's have an equal probability of falling anywhere from 0 to 180 degrees, the probability that 6 or fewer will be greater than 90 degrees is about 5.8×10^{-5}, or about 1 chance in 17,000. Thus it would seem that the A's generally tend to be much smaller than one would expect them to be simply as a result of chance. This tends to negate our first alternate hypothesis, at least in its naive form, and to suggest that the error vectors and the V's are correlated.

Figure 2 (p. 12) illustrates another way of looking for a possible relationship between the angles, Φ, of the error vectors Y-X, and the angles, Ψ, of the

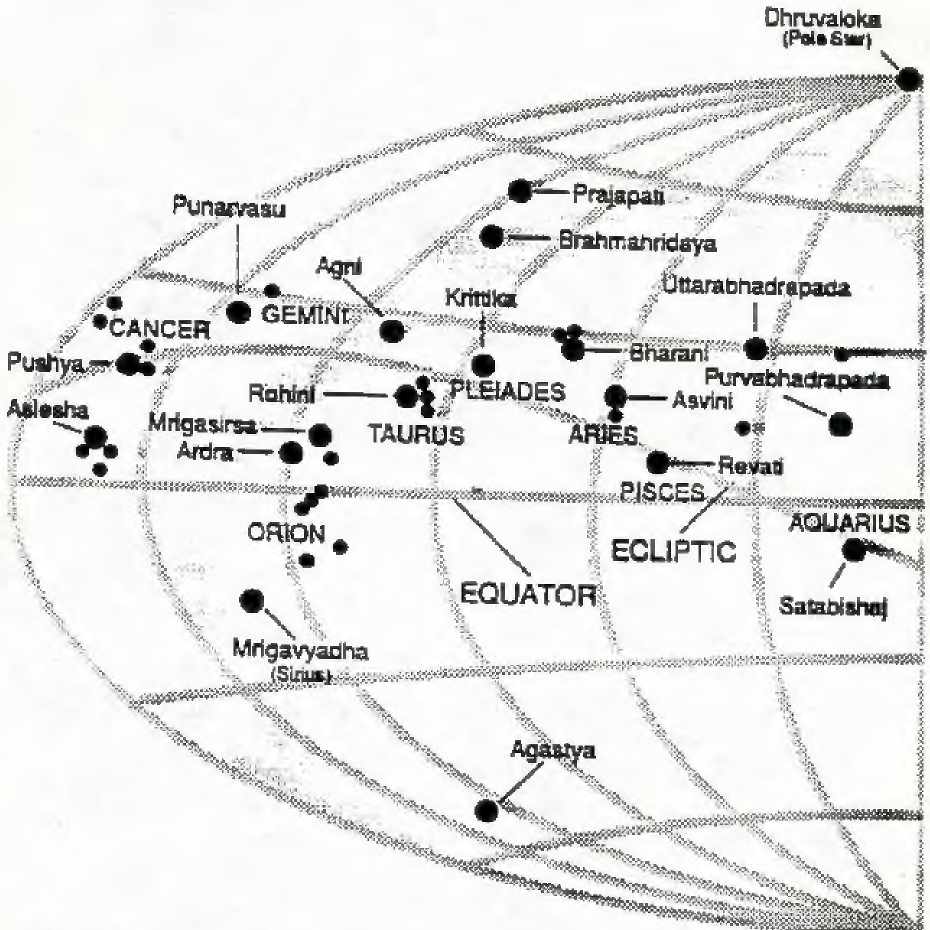


Figure 1. The location of Indian star constellations on the celestial sphere. This figure shows the positions of the stars belonging to the 28 nakshatras, plus the 7 additional stars listed in the *Surya-siddhanta.*

reversed proper motion vectors, V. In this figure, a point, (Φ,Ψ), is plotted for each of the 35 stars of Table 2 (p. 8), and a histogram is plotted of the centered Φ-Ψ differences. Here we say the an angle is "centered" if it is shifted by plus or minus 360 degrees so as to lie between -180 degrees and +180 degrees.

This histogram shows a sharp peak near the center point, where the centered Φ-Ψ equals 0. The angle of approach, A, is simply the absolute value of the centered Φ-Ψ difference, and thus this peak reflects the tendency for the A's to be unexpectedly small. This confirms the hypothesis that the error vectors and

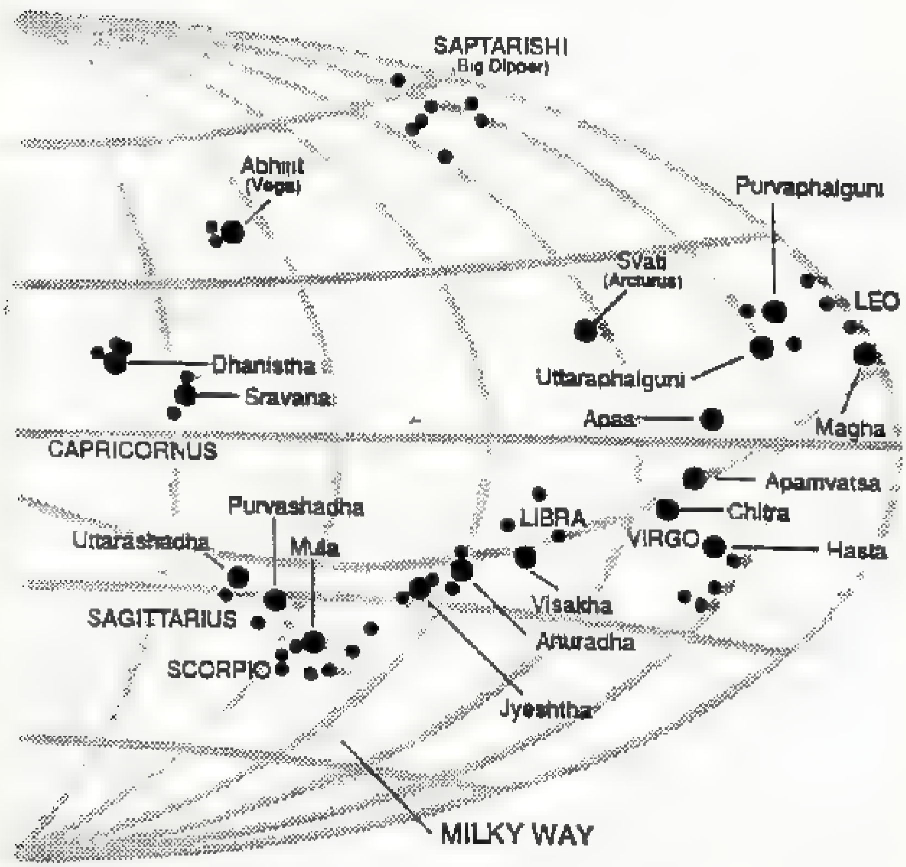


Figure 1—*Continued.* The modern star identifications are obtained from Burgess (1860, pp. 355, 364). The plotted star positions are from a catalogue for A.D. 2000 (Hirshfield and Sinnot 1982).

the V's are significantly correlated. But further examination of Figure 2 gives rise to a doubt.

In this figure we can see that most of the (Φ,Ψ) points are clustered in the upper right hand quadrant of the plot, and this indicates that these angles tend to lie between 0 and 180 degrees. Could it be that this accounts for the apparent correlation between the Φ's and the Ψ's?

Figures 3 and 4 (pp. 12–13) are, respectively, plots of the reversed proper motion vectors, V, and the error vectors, Y-X. We can see that both distributions

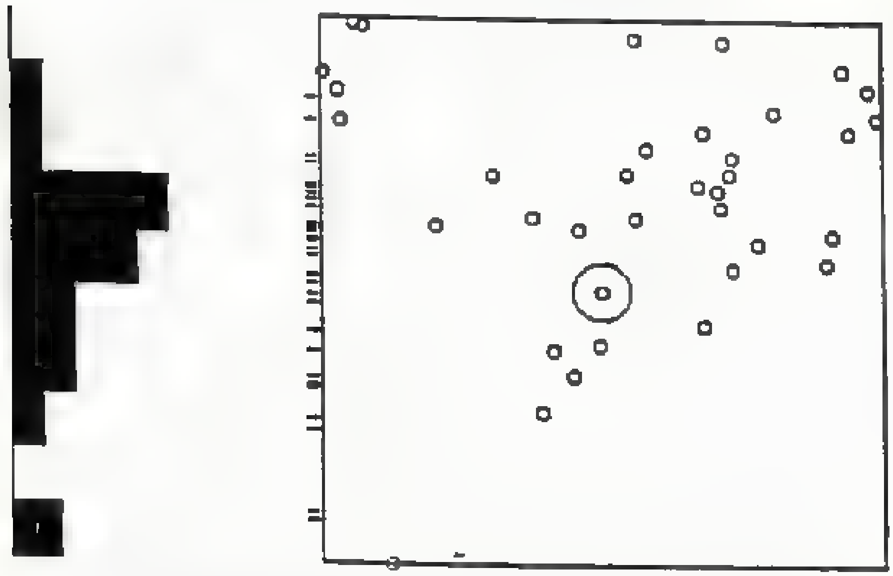


Figure 2. Angle correlation plot of error vectors and reversed proper motion vectors. For each of the 35 stars of table 2 (p. 8), the angle Φ of the vector from X to Y is plotted on the x-axis, and the angle Ψ of V is plotted on the y-axis. All angles range from -180 to 180 degrees. The histogram shows the distribution of the angle differences, Φ-Ψ. To compute this histogram, each Φ-Ψ is expressed as an angle in the range -180 to 180, which is divided into 10 intervals of 36 degrees apiece. The histogram is a plot of the number of Φ-Ψ values falling into each of these intervals.

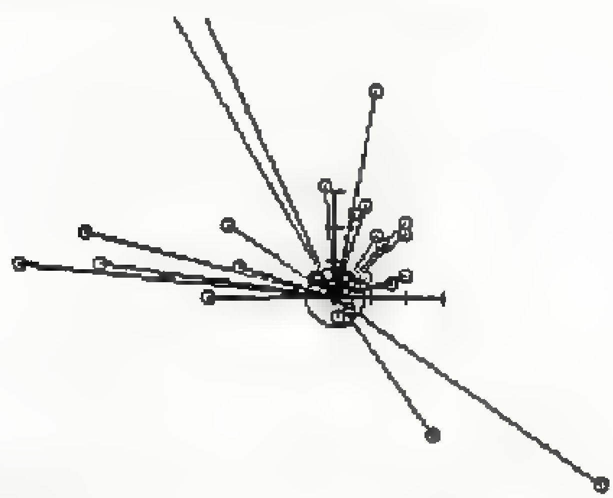


Figure 3. Reversed proper motions—the negatives of the proper motion vectors of the stars in Table 2. The units, marked on the coordinate axes, are in degrees per 50,000 years.

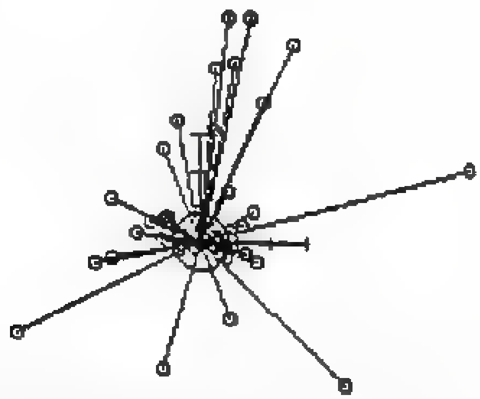


Figure 4. Star coordinate error vectors. These are the vectors from X, the modern star position at A.D. 2000, to Y, the Indian star position precessed from A.D. 490 to A.D. 2000. The stars are as listed in Table 2. The coordinate axes are marked in degrees.

of vectors seem non-random, with a tendency for the vectors to point upwards. This is consistent with what we saw in Figure 2, since a vector pointing upwards has an angle between 0 and 180 degrees. (In Figure 3 the two vectors that run off the page are for the proper motions of Arcturus and Sirius, which are much larger than those of the other stars being considered.)

The non-uniform distribution of the proper motion vectors is explained in textbooks of astronomy as being due to the proper motion of the Sun relative to the nearby stars. However, what can explain the somewhat similar distribution of error vectors Y-X? Do these vectors have a similar distribution because they are correlated with the V's, or do they seem to be correlated with the V's simply because they happen to have a similar distribution for some reason?

We can answer this by comparing the angle Φ for one star with the angle Ψ for another star. We can do this for the $35\times35 = 1225$ pairs of stars that we can form from our star list. For these 1225 pairs of Φ's and Ψ's we can make a histogram of centered Φ-Ψ differences, as before. If the apparent correlation between Φ's and Ψ's is caused by the fact that they tend to be distributed in the interval from 0 to 180 degrees, then this histogram should also show a strong peak in the center.

This is done in Figure 5 (p. 14). The 1225 sets of (Φ,Ψ) points for pairs of stars are plotted in the square, and the resulting histogram is shown on the left. We can see that this histogram does show a slight peak, but this is much less pronounced than the peak obtained in Figure 2 by looking at (Φ,Ψ) pairs with Φ and Ψ belonging to the same star. This indicates that the peak in Figure 2 is mainly due to the fact that each error vector tends to line up with the reversed proper motion vector of its own star.

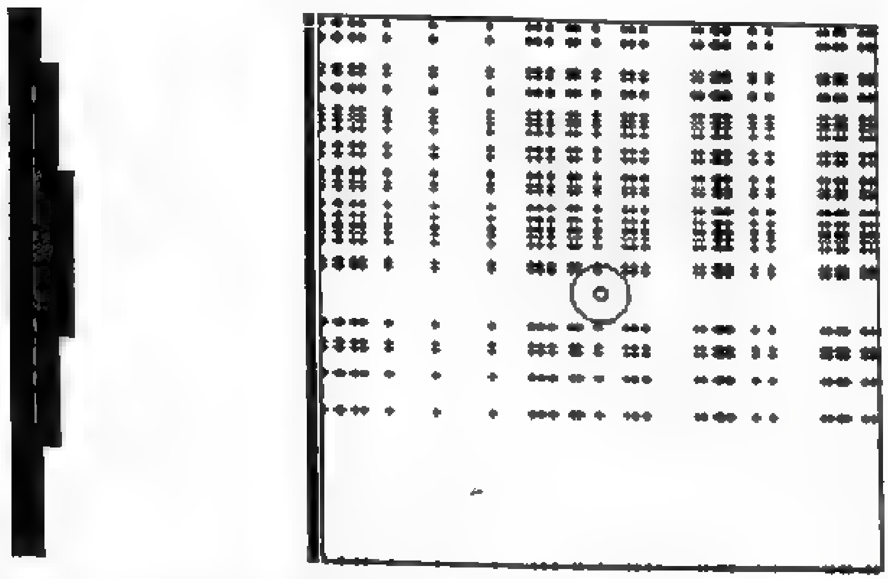


Figure 5. Angle correlation plot for scrambled error vectors and reversed proper motion vectors. Here we plot the angle Φ for one star from Table 2 (p. 8) on the x-axis, and the angle Ψ for another star on the y-axis. This is done for 35×35 = 1225 pairs of stars. The resulting histogram shows to what extent the uneven distributions of the Φ's and Ψ's may tend to create an apparent correlation between these angles. (See the text.)

2. INDIVIDUAL EXAMINATION OF STAR COORDINATES

Our examination of the distribution of angles between error vectors and reversed proper motion vectors tends to indicate that there is some relation between these two sets of variables. However, it does not tell us what the reason for that relation might be, and in particular, it does not shed much light on our hypothesis of great antiquity for the Indian star coordinates. All we can say thus far, is that a relationship is there, and our hypothesis does imply that such a relationship should exist.

To try to learn more, we will turn to the examination of the star coordinates on an individual basis. Table 3 is the same as Table 2 (p. 8), with two exceptions. First, certain star identifications made by Burgess have been modified on the basis of explicit examination of star positions and motions. We will carry out this examination in the main body of this section.

Second, a set of 8 stars from Svati to Uttaraphalguni has been moved to the bottom of the table. These are the stars for which the ratio D(5)/D is greater than 1.9. As we will show, a study of the 35 sets of star coordinates reveals a number of cases in which X(t) comes closest to Y when t is near 5. This tends to suggest

TABLE 3

Indian Star Coordinates Compared with Modern Coordinates

No.	Sanskrit Name	Modern Name	D	D(5)	A	Magn.
11	Purvaphalguni	*Theta Leonis	1.881	.429	.542	3.34
32	Brahmahridaya	Alpha Aurigae	6.487	531	4.011	.08
29	Agastya	Alpha Carinae	.846	.377	7.447	-.72
4	Rohini	Alpha Tauri	1.650	1.579	8.524	.85
3	Krittika	Eta Tauri	.613	.114	8.750	2.87
5	Mrigasirsa	>Phi2 Orionis	5.164	.901	9.204	4.09
13	Hasta	Delta Corvi	2.839	.942	9.895	2.95
19	Mula	>Epsilon Scorpionis	8.823	1.696	10.935	2.29
8	Pushya	*Theta Cancri	1.819	.765	16.045	5.35
33	Prajapati	Delta Aurigae	6.571	4.687	22.397	3.72
1	Asvini	Beta Arietis	1.271	1.043	27.117	2.64
35	Apas	Delta Virginis	5.910	3.615	33.833	3.38
14	Chitra	Alpha Virginis	.943	.582	38.093	.98
34	Apamvatsa	Theta Virginis	1.844	1.374	39.022	4.38
25	Satabhisaj	Lambda Aquarii	.673	.435	40.104	3.74
18	Jyestha	Alpha Scorpionis	1.499	1.326	53.067	.96
2	Bharani	*41 Arietis	2.693	2.275	56.667	3.63
16	Visakha	24 Iota Librae	2.586	2.290	59.600	4.54
31	Agni	Beta Tauri	7.925	7.189	63.972	1.65
24	Dhanishtha	Beta Delphini	3.203	2.979	67.581	3.54
26	Purvabhadrapada	Alpha Pegasi	3.459	3.226	68.341	2.49
17	Anuradha	Delta Scorpionis	3.053	3.026	82.294	2.32
9	Aslesha	Epsilon Hydrae	5.230	5.774	85.337	3.38
6	Ardra	Alpha Orionis	6.261	6.251	86.837	.50
27	Uttarabhadrapada	Alpha Andromedo	5.779	6.524	92.205	-.16
20	Purvashadha	Delta Sagittarii	1.251	1.604	111.208	2.70
21	Uttarashadha	Sigma Sagittarii	2.306	2.976	146.561	2.02
15	Svati	Alpha Bootis	2.851	25.574	8.638	-.04
7	Punarvasu	Beta Geminorum	.689	8.189	26.553	1.14
23	Sravana	Alpha Aquilae	.709	6.660	32.233	.77
22	Abhijit	Alpha Lyrae	1.680	3.230	36.023	.03
10	Magha	Alpha Leonis	.214	3.336	43.313	1.35
30	Mrigavyadha	Alpha Canis Major	1.119	15.133	62.610	-1.46
28	Revati	Zeta Piscium	1.182	2.989	125.959	4.86
12	Uttaraphalguni	Beta Leonis	.411	7.415	145.010	2.14

This table is the same as Table 2, with two exceptions. First, certain star identifications made by Burgess have been modified on the basis of explicit examination of star positions and motions. Modern star names marked with (*) are alternative yogataras mentioned by Burgess; names marked (>) are stars from the same nakshatra constellations which Burgess did not mention as possible yogataras. With these modifications, the average angle of approach, A, is 49.34 degrees. Second, a group of 8 stars from Svati to Uttaraphalguni has been moved to the bottom of the table. These are the stars for which the ratio D(5)/D is greater than 1.9.

50,000 B.E. as the date for these coordinates. However, for the 8 stars moved to the bottom of the table, X(t) comes closest to Y at much smaller values of t. By t=5 these stars have moved far away from the corresponding Indian star positions, and thus D(5)/D tends to be large. We will first discuss the 27 cases in the upper part of the table in three groups of 9, and then we will separately discuss the 8 cases with large D(5)/D values.

Group 1. The first group of nine cases consists of stars that move almost directly towards the corresponding Indian star position as we move back in time. For these stars the value of the angle of approach in Table 3 varies from 0.542 to 16.045 degrees, and D(5)/D tends to be quite small (except in the case of Rohini—see below). The date of minimum approach between the modern star and the Indian star position averages out to 54,119 B.E. for these nine cases, and the standard deviation of these dates is 18,415 years.

In discussing these cases, we will refer to the opinions of Colebrooke (1807) and Burgess (1860) on star identifications. Their remarks convey some idea of how reliable these identifications are, and this has important bearing on how much weight we can give to these identifications in the course of our analysis.

In each case, we present a figure showing the relevant star positions and movements, followed by a discussion of that case. In the figures, the modern stars are shown at A.D. 2000 and at eight 10,000 year intervals preceding this date. Thus, each modern star is represented by a track that begins with an enlarged circle at the star's position in A.D. 2000, and moves by smaller circles through successive past positions. The modern star selected as the yogatara is represented by larger circles.

The Indian star position is precessed to A.D. 2000 from A.D. 490+50n, where n runs from -8 to 8. The resulting positions are plotted in the figures, with the position corresponding to A.D. 490 (n=0) at the origin. The result is a nearly horizontal sequence of circles representing different possible epochs for the Indian star coordinates. This makes it possible to see from examination of the figures the effect of changing this epoch from 490 to some other value.

The x-axis corresponds to the celestial circle of latitude passing through the Indian star coordinates precessed from A.D. 490, and the y-axis corresponds to the celestial meridian through this point. The units for both axes are degrees as measured at the celestial equator. The scale varies from figure to figure, depending on the sizes of the various constellations, but the units are marked on the coordinate axes.

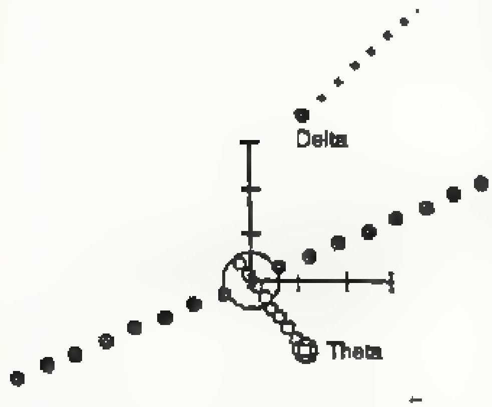


Figure s1. Purvaphalguni (11). Here both Colebrooke and Burgess agree that the nakshatra consists of Delta and Theta Leonis. The *Surya-siddhanta* indicates that the yogatara is the northern star, and thus is Delta Leonis. However, Colebrooke suggests that Brahmagupta and Bhascara selected the southern star, Theta, as the yogatara, and Burgess makes a similar comment.

In the figure we can see that Theta Leonis was very close to the yogatara position about 60,000 years ago. In fact, it was at a distance of about 0.018 degrees in 64,430 B.C. Delta, however, moves further from this position as we go back in time.

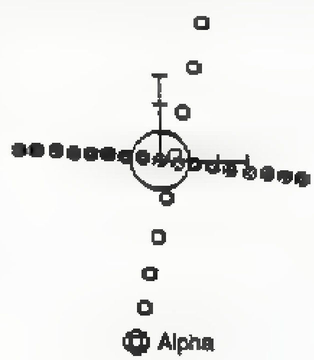


Figure s2. Brahmahrdaya (32). Both Colebrooke and Burgess agree that this star is Capella (Alpha Aurigae). Colebrooke also cites native testimony in support of this.

The figure shows that Capella was very close to the position of Brahmahrdaya about 50,000 years ago. Its distance will be 6.487 degrees in A.D. 2000, and it reached a minimum value of 0.453 degrees in 48,164 B.C.

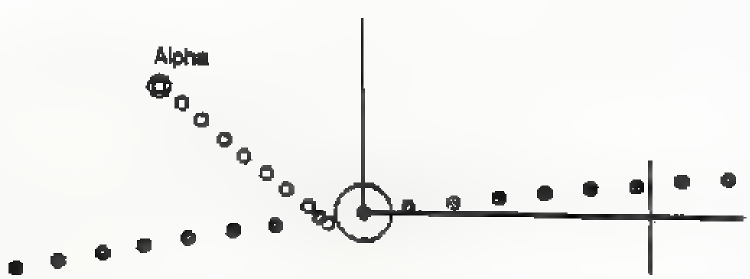


Figure s3. Agastya (29). Colebrooke and Burgess agree that Agastya is Canopus (Alpha Carinae). This star was at a minimum of about 0.110 degrees from Agastya's position in 87,490 B.C., and was 0.377 degrees away in 50,000 B.C.

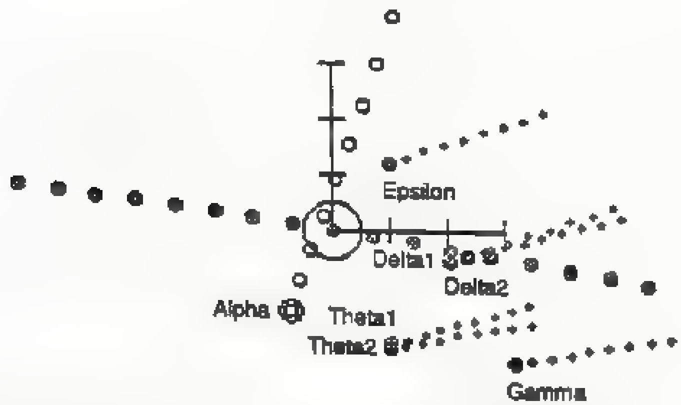


Figure s4. Rohini (4). Colebrooke and Burgess agree that Aldebaran (Alpha Tauri) is the yogatara of this nakshatra. Burgess and Kay list its other stars as Theta Gamma Delta Epsilon Tauri. Aldebaran is the eastern star of this group, as required by the *Surya-siddhanta*.

As we go back in time, Aldebaran moves toward Rohini at an angle of 8.524 degrees. It is closest to Rohini at a distance of 0.245 degrees in 27,295 B.E.

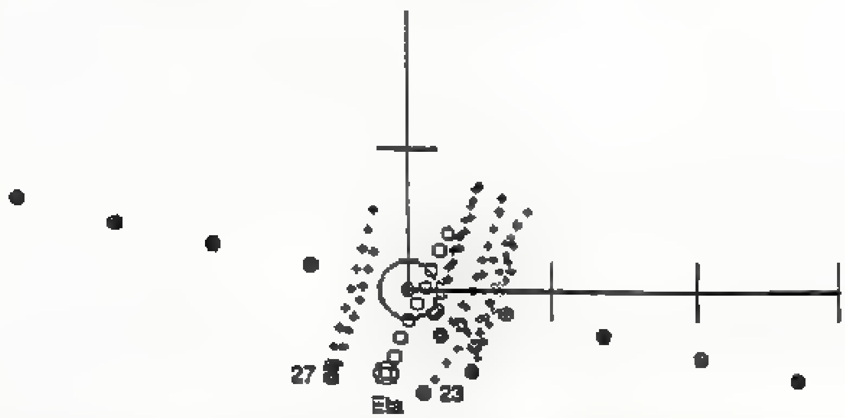


Figure s5. Krittika (3). Colebrooke and Burgess agree that the nakshatra *Krittika* consists of six stars of the Pleiades. These six stars are associated with six wives of the Sapta Rishis (the seven sages) who became foster mothers of Kartikeya. (We show more than six stars, since it is hard to know which six were intended.)

Burgess notes that the southern star of this group should be the yogatara, according to the *Surya-siddhanta*, and this would be Atlas (27 Tauri) or Merope (23 Tauri). However, Alcyone (Eta Tauri) is the brightest in the group, and he concludes that it must be the yogatara.

In A.D. 2000 Alcyone is .613 degrees from the yogatara position, but in 45,158 B.E. this distance was at a minimum of 0.093 degrees.

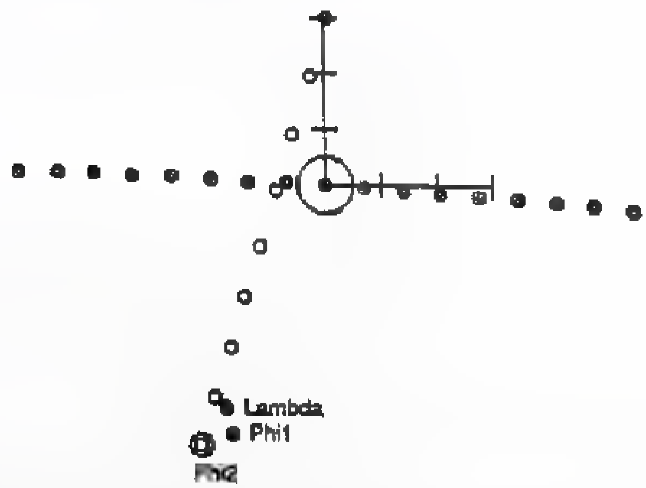


Figure s6. Mrigasirsa (5). Colebrooke and Burgess agree that this nakshatra must consist of the three stars in the head of Orion (Lambda Phi1 Phi2 Orionis). Of these, the northern star should be the yogatara according to *Surya-siddhanta*, and thus is Lambda.

It turns out that Lambda Orionis barely moves from its position in a period lasting 50,000 years. However, as we go back in time, Phi2 Orionis moves towards the Mrigasirsa position at an angle of 9.204 degrees, and it was at a minimum distance of 0.825 degrees in 53,500 B.E.

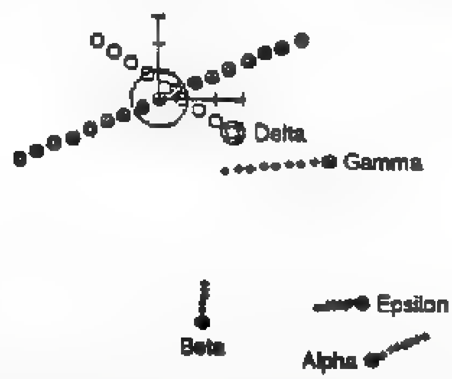


Figure s7. Hasta (13). Colebrooke and Burgess agree that this nakshatra consists of Alpha Beta Gamma Delta Epsilon Corvi. Burgess points out that the description in *Surya-siddhanta* of the yogatara is ambiguous, but seems to indicate Gamma. He then goes on to say that "the defined position, in which all authorities agree, would point rather to Delta" (Burgess, p. 334).

Delta Corvi was at a minimum distance of 0.488 degrees from the position of Hasta in 38,912 B.E. The figure shows that none of the other stars in this group tend to closely approach the yogatara position over the last 80,000 years.

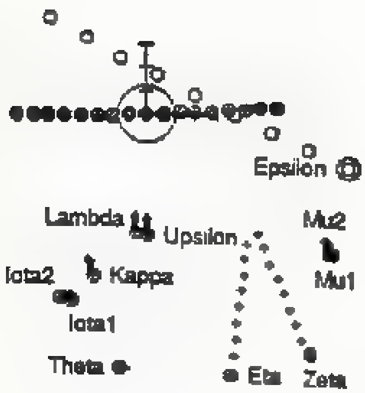


Figure s8. Mula (19). Colebrooke and Burgess agree that this nakshatra should consist of the stars Upsilon Lambda Kappa Iota Theta Eta Zeta Mu Epsilon Scorpionis that form the tail of the scorpion. Sakalya says that there are nine stars in this nakshatra, and other authorities say that there are eleven. Here they may be noting the fact that Mu and Zeta are closely spaced double stars. (Iota is also double.)

The *Surya-siddhanta* says that the yogatara is the eastern star of the group, and this is Iota. However, Colebrooke selects Upsilon and Burgess selects Lambda. This is done on the basis of position and agreement with the Arabic manzil ash-Shaulah, which consists of Upsilon and Lambda.

It turns out that Upsilon and Lambda barely move over a 50,000 year period. However, the figure shows that the star Epsilon reached a minimum distance of 1.667 degrees from the position of Mula in 48,223 B.C.

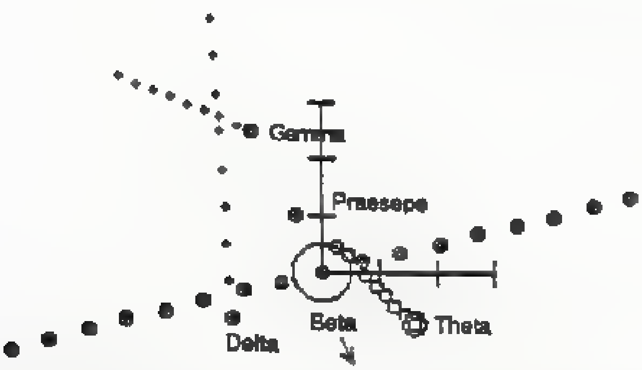


Figure s9. Pushya (8). Colebrooke regards this nakshatra as consisting of Delta Gamma Beta Cancri, and Burgess chooses Delta Gamma Theta or Beta Gamma Theta and the nebulous cluster, Praesepe. The middle star is the yogatara, according to the *Surya-siddhanta*, and this would be Delta for the first Burgess selection and Theta for the second.

Burgess notes that Theta is now dim (magnitude 5.35), but Ptolemy lists it as having a magnitude of 4. In general, it appears that the luminosity of stars can vary considerably over thousands of years.

As we go back in time, Theta moves from a distance of 1.819 degrees from Pushya in A.D. 2000 to 0.765 degrees in 50,000 B.C. Its minimum, 0.503 degrees, is in 73,900 B.C.

Group 2. The next group of nine cases consists of stars that move roughly in the direction of the corresponding Indian star position as we move back in time. For these stars the value of the angle of approach in Table 3 varies from 22.397 to 59.6 degrees, and D(5)/D varies from 0.612 to 0.886. There are several dates of closest approach near 50,000 B.E., but also several that are much larger.

Figure s10. Prajapati (33). Colebrooke and Burgess agree that Prajapati is Delta Aurigae. As we can see in the figure, this star approaches the position of Prajapati as we move back in time, but it takes 142,119 years before it reaches its minimum distance of 2.500 degrees. At 50,000 B.E. the distance is still 4.687 degrees.

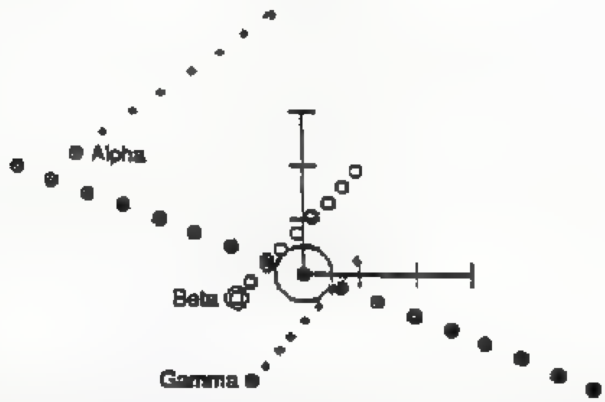


Figure s11. Asvini (1). Burgess concludes that this nakshatra consists of Beta Gamma Arietis (and maybe Alpha Arietis). According to the *Surya-siddhanta*, the yogatara is the northern star; Colebrooke selects Alpha for the yogatara, but Burgess chooses Beta on the basis of position and shastric references saying that Asvini consists of just two stars.

Beta Arietis is 1.271 degrees from the position of Asvini in A.D. 2000, and it reached a minimum distance of 0.580 degrees in 28,213 B.E. Gamma also came close to Asvini's position in the past, reaching a minimum distance of 0.567 degrees in 57,782 B.E.

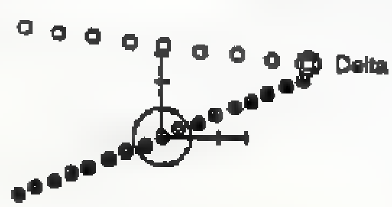


Figure s12. Apas (35). Colebrooke and Burgess agree that this star is Delta Virginis. Going back in time, Delta Virginis went from a distance of 5.910 degrees from Apas in A.D. 2000 to a minimum distance of 3.286 degrees in 38,018 B.E.

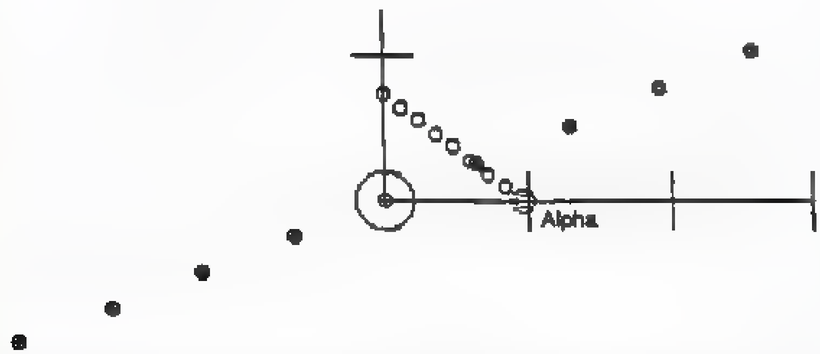


Figure s13. Chitra (14). Colebrooke and Burgess agree that this naksatra consists of the single star Spica (Alpha Virginis). It was situated at a minimum distance of 0.582 degrees from the naksatra position in 49,477 B.B.

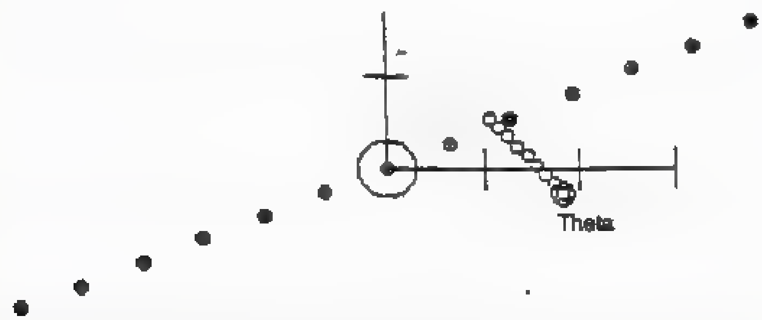


Figure s14. Apamvatsa (34). Burgess identifies this star as Theta Virginis, and Colebrooke's identification is unclear. This star was closer to the position of Apamvatsa in 50,000 B.B., and it reached a minimum distance of 1.161 degrees in 102,955 B.B.

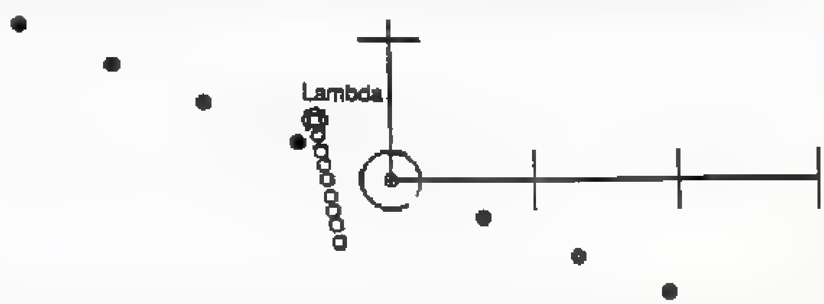


Figure s15. Satabhisaj (25). Colebrooke and Burgess agree that this nakshatra consists of 100 stars in the stream from the Jar or in the right leg of Aquarius. They identify the yogatara as Lambda Aquarii. This star reached its minimum distance from the yogatara position at 0.434 degrees in 46,966 B.B.

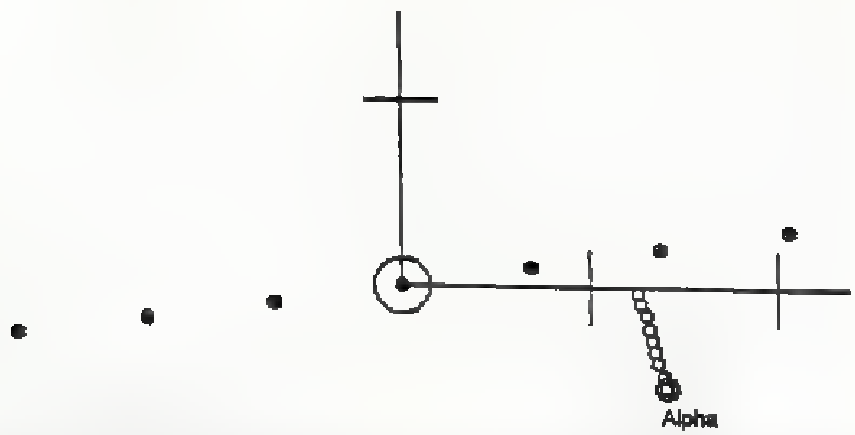


Figure s16. Jyestha (18). Colebrooke and Burgess agree that this nakshatra consists of Alpha Sigma Tau Scorpionis, and that the yogatara is Antares (Alpha Scorpionis). Antares was 1.499 degrees from Jyeṣṭha's position in A.D. 2000, and it reached a minimum distance of 1.200 degrees in 135,944 B.E.

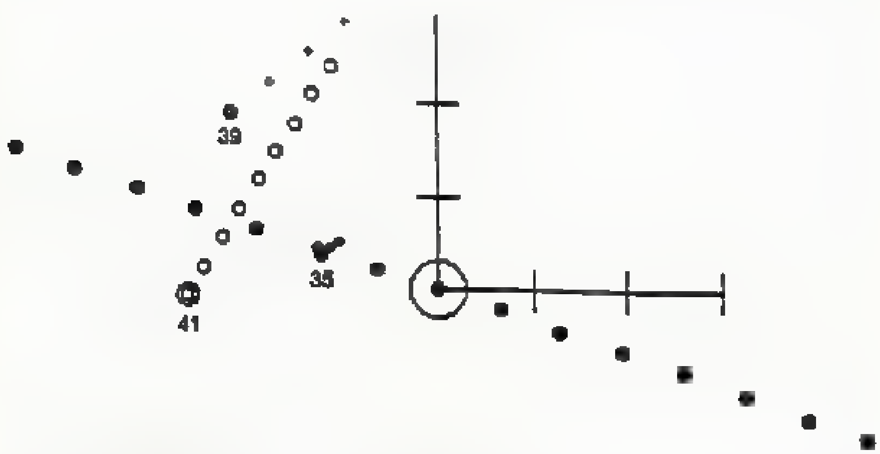


Figure s17. Bharani (2). Colebrooke cites testimony indicating that this nakshatra consists of three stars in Musca Borealis. Burgess identifies these as 35 41 39 Arietis. The yogatara is stated to be the southern star of the group in the *Surya-siddhanta*, and this should be 41. Burgess also suggests 35 on the basis of position.

It turns out that 35 Arietis barely moves in the last 50,000 years. However, 41 Arietis reaches a minimum distance of 2.250 degrees from the yogatara's position in 40,710 B.E. Since the angle of approach is 56.667 degrees, this represents only a modest decrease in the original distance in A.D. 2000.

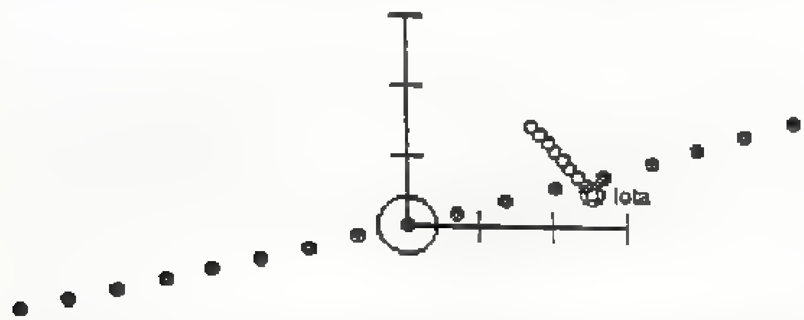


Figure s18. Visakha (16). There is considerable confusion in the identification of the stars making up this nakshatra. Colebrooke concludes that the stars are Alpha Nu Iota Librae and Gamma Scorpionis, with Alpha Librae as the yogatara. Burgess disagrees and chooses Iota Alpha Beta Gamma Librae. He argues on the basis of position that Iota should be the yogatara. Since some authorities say that this nakshatra should have two rather than four stars, and the yogatara should be the northern star, he also suggests that Visakha might consist of Iota Librae and 20 Librae to the south. At this point he complains that, "The whole scheme of designations we regard as of inferior authenticity, and as partaking of the confusion and uncertainty of the later knowledge of the Hindus respecting their system of asterisms."

As we go back in time, Iota Librae moves in the direction of the yogatara position at an angle of 59.600 degrees, reaching a minimum distance of 2.231 degrees in 82,960 B.B.

Group 3. The next group of nine cases consists of stars that do not tend to move strongly toward the corresponding Indian star positions as we move back in time. However, all but three of these stars do move slightly towards these positions. The angles of approach range from 63.972 degrees to 86.837 degrees for the first six of these cases. The last three move away, with angles of approach greater than 90 degrees. In the nine cases the ratio of D(5)/D ranges from 0.907 to 1.291.

Since these stars tend to move roughly perpendicular to the direction of the Indian star position, their dates of minimum approach are not very meaningful, and we omit them. (Of course, this cutoff at about 60 degrees in angle of approach is somewhat arbitrary.)

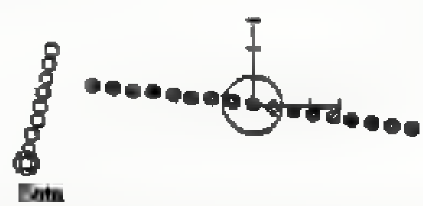


Figure s19. Agni or Hutabhuj (31). Both Colebrooke and Burgess regard this star as Beta Tauri.

Figure s20. Sravishtha or Dhanishtha (24). Colebrooke reports testimony from Jesuits in India indicating that this nakshatra should consist of Alpha Beta Gamma Delta Delphini. Burgess agrees and notes that the yogatara should be the westernmost star, according to *Surya-siddhanta*. This should be Beta, although Burgess also suggests Zeta Delphini as a possibility.

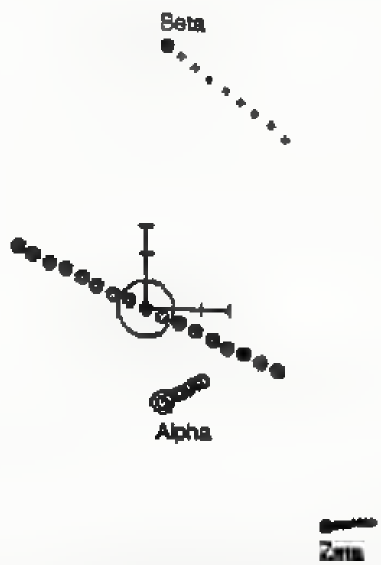


Figure s21. Purvabhadrapada (26). Both Colebrooke and Burgess argue that this nakshatra should consist of Alpha Zeta Pegasi or perhaps Alpha Beta Pegasi. In either case they choose Alpha as the yogatara, and in the first case this will be the northern star, as required by the *Surya-siddhanta*. All three of these stars are shown in the figure.

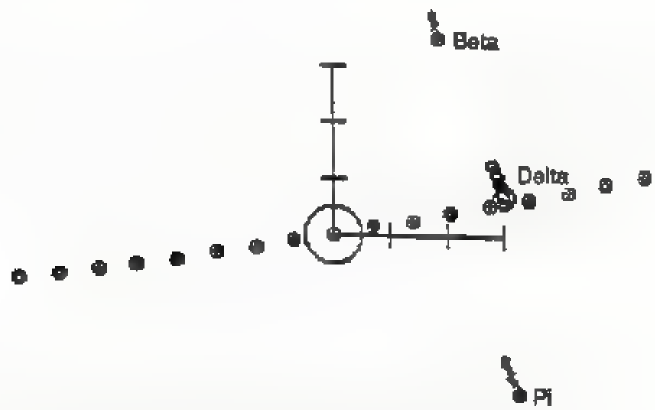


Figure s22. Anuradha (17). Both Colebrooke and Burgess agree that this nakshatra should contain Beta Delta Pi Scorpionis, with Delta as the yogatara. Rho Scorpionis may also be included.

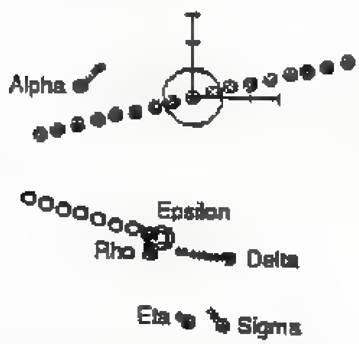


Figure s23. Aslesha (9). Colebrooke argued that this nakshatra should be Alpha Cancri plus four other unidentified stars. Burgess pointed out that suitable stars were not to be found near Alpha Cancri, and suggested that the nakshatra should consist of Epsilon Eta Sigma Delta Rho Hydrae, with Epsilon as the yogatara. All of these stars are shown in the figure.

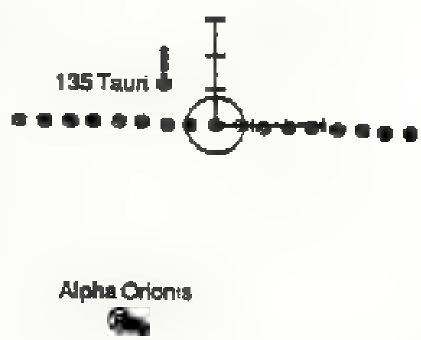


Figure s24. Ardra (6). Both Colebrooke and Burgess agree that this nakshatra should consist of the single star Betelgeuse (Alpha Orionis). However, Burgess is dissatisfied, since Betelgeuse is 6.261 degrees from the position of Ardra. He suggests a dim star, 135 Tauri that is closer to this position, but notes that neither star is of the third magnitude, as required by the verse in *Surya-siddhanta* describing the heliacal rising of Ardra. He concludes that, "We confess ourselves unable to account for the confusion existing with regard to this asterism, of which Albiruni also could obtain no intelligible account from his Indian teachers."

Figure s25. Uttarabhadrapada (27). Both Colebrooke and Burgess agree that this nakshatra should consist of Alpha Andromedo and Gamma Pegasi. Colebrooke chooses Alpha Andromedo as the yogatara.

However, Burgess suggests that Alpha and Gamma Pegasi were originally the southern junction stars of the two Bhadrapadas, (see Purvabhadrapada above) and that the rank of junction star was for some reason transferred to the northern stars in the two asterisms. He says, "in making the transfer, the original constitution of the former group [Purvabhadrapada] was neglected, while in the latter [Uttarabhadrapada] the attempt was made to define the real position of the northern star, but by simply adding to the polar latitude already stated for Gamma Pegasi, without altering its polar longitude also." Thus, he maintains, Uttarabhadrapada was assigned the longitude of Gamma Pegasi and the latitude of Alpha Andromedo.

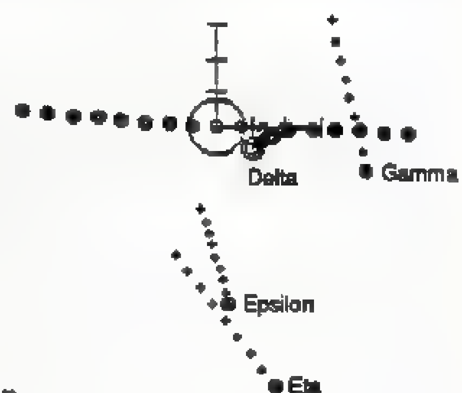


Figure s26. Purvashadha (20). Both Colebrooke and Burgess agree that if two stars are intended for this nakshatra, these should be Delta Epsilon Sagittarii, with Delta as the yogatara. Sakalya assigns four stars to this nakshatra, and Burgess proposes that these should be Gamma2 Delta Epsilon Eta Sagittarii; this is also stated by Albiruni.

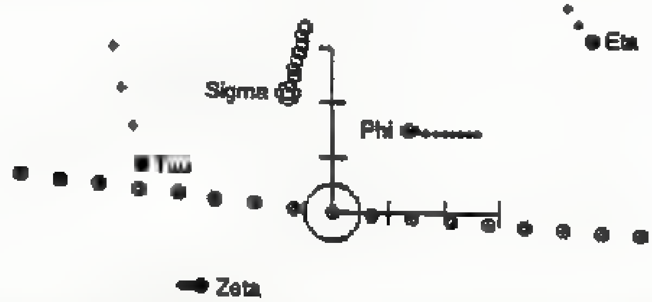


Figure s27. Uttarashadha (21). Colebrooke assigns the two stars Tau Zeta Sagittarii to this nakshatra, with Tau as the yogatara. Burgess assigns Sigma Zeta Sagittarii, with Sigma as the yogatara. Sakalya assigns 4 stars to Uttarasadha, and Burgess suggests that these should be Phi Sigma Tau Zeta Sagittarii; this is also stated by Albiruni. We also note that M. Biot assigns Tau Sagittarii as the yogatara.

Group 4. Unlike the last group, the following group of 8 stars contains several examples that move towards the corresponding Indian star positions at small angles of approach. However, these stars tend to be either fast moving, or initially close to the Indian star position, or both. Thus, they tend to move rapidly past that position as we go back in time, and therefore they tend to have high D(5)/D ratios. The group is defined by the criterion that D(5)/D>1.9. Apart from Revati, which is a special case, these stars also tend to be very bright.

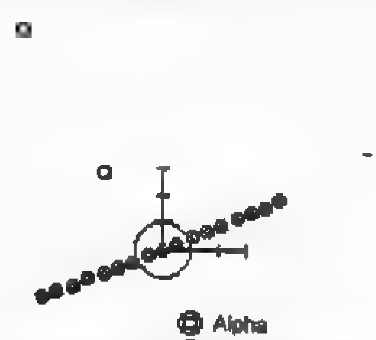


Figure s28. Svati (15). Colebrooke and Burgess agree that this nakshatra consists of the single star Arcturus (Alpha Bootis), and Colebrooke cites native testimony supporting this.

As we go back in time, Arcturus approaches the position of Svati at an 8.638 degree angle, reaching a minimum distance of 0.428 degrees in 4460 B.E. The distance in A.D. 2000 is 2.851 degrees.

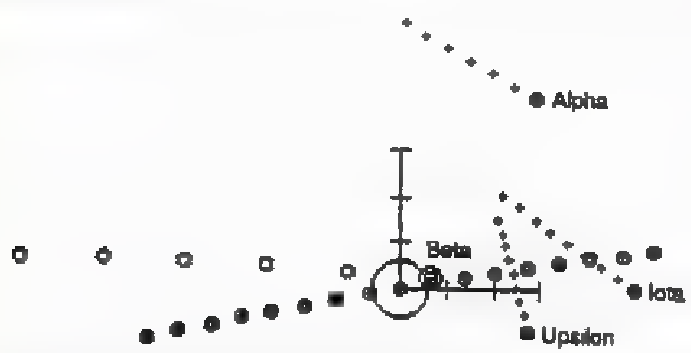


Figure s29. Punarvasu (7). Some authorities assign two stars to this nakshatra, and Colebrooke and Burgess agree that these should be Alpha Beta Geminorum. Others say that there are four stars, and thus Colebrooke adds Theta Tau Geminorum and Burgess adds Iota Upsilon Geminorum. The yogatara should be to the east according to the *Surya-siddhanta*, and this star is Beta Geminorum.

As we go backward in time, Beta Geminorum approaches the yogatara position at a 26.553 degree angle, reaching a minimum distance of 0.308 degrees in 3520 B.E.

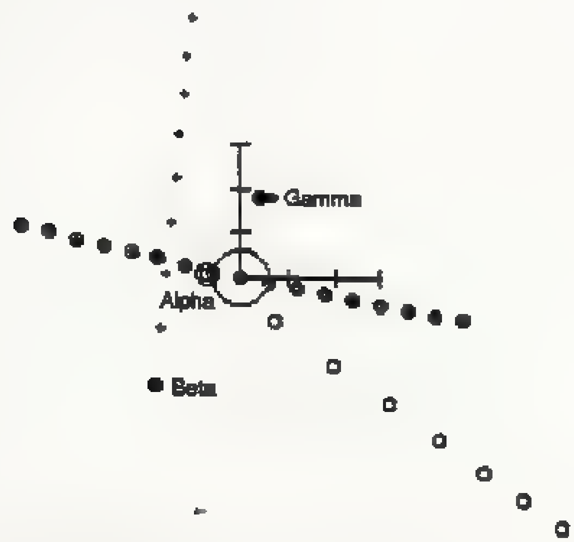


Figure s30. Sravana (23). Colebrooke and Burgess agree that this nakshatra consists of Alpha Beta Gamma Aquilae, with Alpha as the yogatara.

As we go back in time, this star approaches the yogatara position at a 32.233 degree angle, reaching a minimum distance of 0.378 degrees in 3318 B.C.

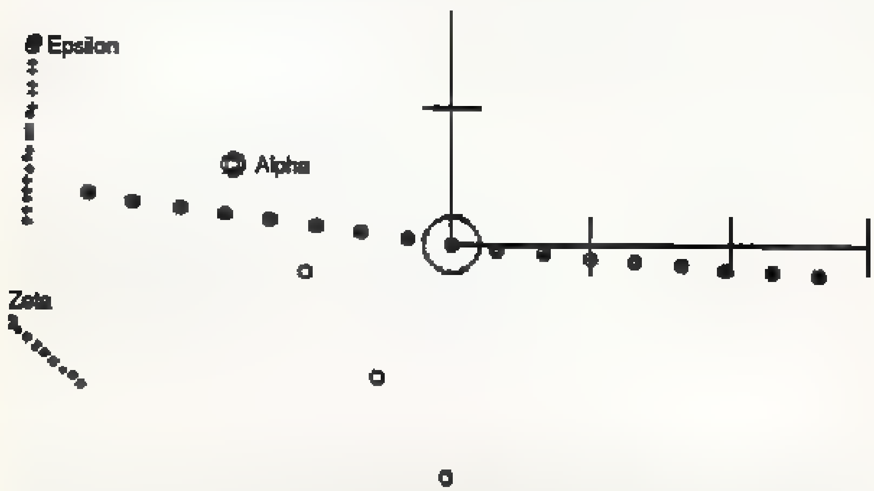


Figure s31. Abhijit (22). Colebrooke and Burgess agree that the yogatara of this nakshatra is Vega (Alpha Lyrae), and Burgess adds the stars Epsilon Zeta Lyrae.

As we go back in time, Vega approaches the yogatara position at a 36.023 degree angle, reaching a minimum distance of 0.990 degrees in 14,409 B.C. The distance in the year A.D. 2000 is 1.680 degrees.

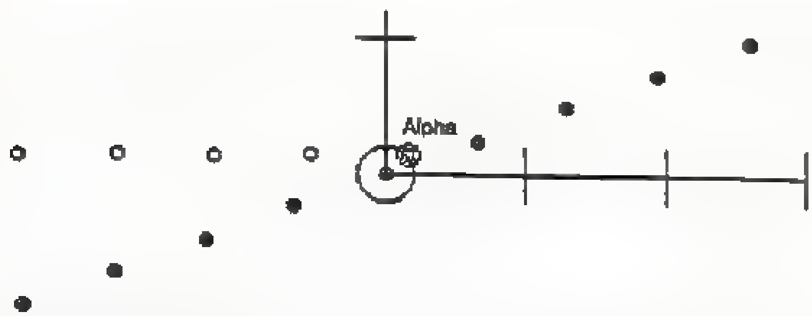
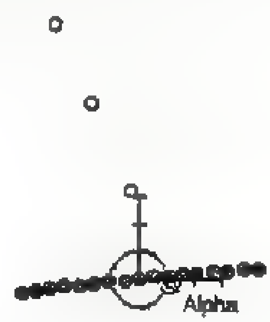


Figure s32. Magha (10). Colebrooke and Burgess agree that Regulus (Alpha Leonis) is the yogatara of this nakshatra, but they express some indecision regarding the remaining stars in the group. Their best suggestion seems to be that this nakshatra corresponds to the constellation known as the Sickle: Alpha Eta Gamma Zeta Mu Epsilon Leonis.

As we go back in time, Regulus approaches the yogatara position at a 43.313 degree angle, reaching a minimum distance of 0.147 degrees in 2250 B.C.

Figure s33. Mrigavyadha (30). Colebrooke and Burgess agree that this star is Sirius (Alpha Canis Major). Going back in time, Sirius approaches the position of Mrigavyadha at a 62.610 degree angle, reaching a minimum distance of 0.994 degrees in 1400 B.C. The distance at A.D. 2000 is 1.119 degrees, and thus Sirius only roughly approaches Mrigavyadha.



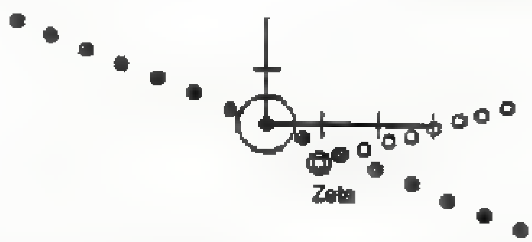


Figure s34. Revati (28). Colebrooke, Burgess, and most other authorities on Indian astronomy agree that the yogatara of this nakshatra should be Zeta Piscium. There are said to be 31 other stars in this nakshatra, but their identity seems to be unknown. The yogatara of Revati is important, since it is used as the starting point for measurements of celestial longitude in ancient Indian astronomy. However, it is a dim star (of magnitude 4.86) and is one of those which Albiruni was unable to identify.

As we go back in time, Zeta Piscium moves sharply away from the yogatara position at an angle of 125.959 degrees.

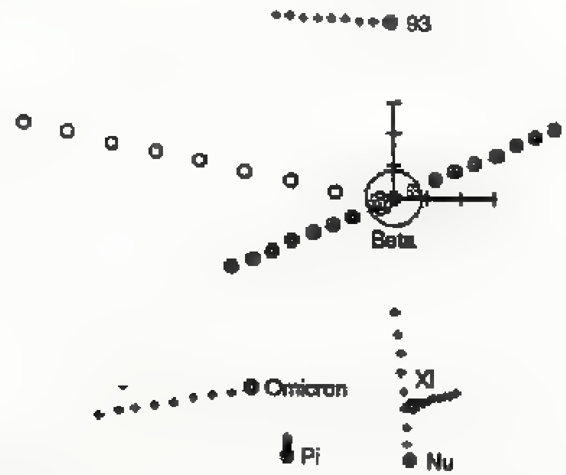


Figure s35. Uttaraphalguni (12). Colebrooke and Burgess agree that Beta Leonis should be the yogatara of this nakshatra, but there is some confusion regarding the other stars. If there are two in total, Burgess suggests 93 Leonis for the other. However, this is to the north of Beta, and the *Surya-siddhanta* says that the yogatara should be the northern star. Sakalya gives the nakshatra five stars, and Burgess suggests that these might be Beta Leonis and probably Xi_1 Nu Pi Omicron Virginis.

As we go back in time, Beta Leonis moves away from the yogatara position at an angle of 145.010 degrees.

In this survey of the 35 cases we have obtained 35 estimates of the age of the Indian star coordinates (although some were not mentioned since the large angle of approach renders them meaningless). For each Indian star position, Y, and corresponding modern star position, X, this age estimate is the time T at which X(T) comes closest to Y. One way of evaluating these estimates is to simply use all of them to plot a histogram, as is shown in Figure 6 (p. 32). In this histogram, the bars represent the number of times falling into different time intervals. Starting on the left, the first interval is from 0 B.E. back to 5000 B.E., and the succeeding intervals cover 10,000 years apiece.

The first interval shows a pronounced peak representing the stars in Table 3 (p. 15) beginning with Svati, which have times of closest approach of a few thousand years. There is also a strong peak in the 6th interval, which ranges from 45,000 B.E. to 55,000 B.E. It appears from this evidence that the 35 sets of star coordinates divide naturally into a relatively young population and a much older population dating back to about 50,000 B.E.

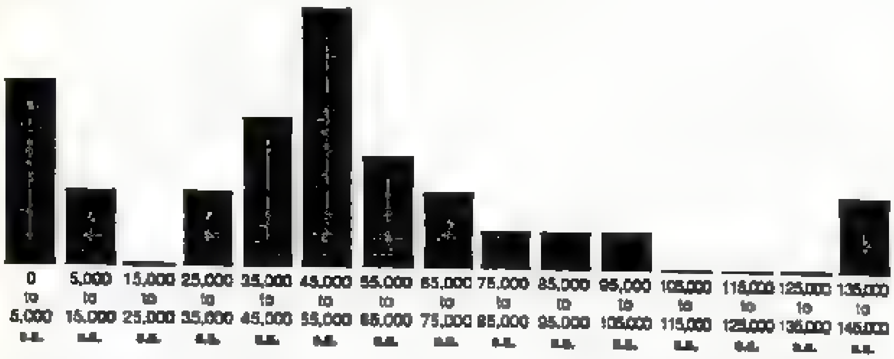


Figure 6. Histogram of age estimates for the star coordinates and identifications used in Table 3. For each Indian star position, and corresponding star from modern catalogues, we can compute the time in the past when that star was closest to the Indian position. In the histogram, the bars represent the number of times falling into different time intervals. Starting on the left, the first interval is from 0 B.E. back to 5000 B.E., and the succeeding intervals cover 10,000 years apiece.

We see strong peaks in the interval from 0 B.E. to 5000 B.E. and in the interval from 45,000 B.E. to 55,000 B.E.

3. STELLAR MOTIONS, THE IDENTIFICATION OF THE NAKSHATRAS, AND CROSS-CULTURAL RELATIONSHIPS

Table 4 (p. 35) displays some supplementary information which is available for the 28 nakshatras, but not for stars 29–35. In this table, the 28 nakshatras are listed in the same order in which they appear in Table 3 (p. 15), and they are divided into the four groups discussed in connection with Figures s1–s35. The column under Alt lists the number of alternative opinions regarding the identity of the yogataras found in Dikshit's book, *Bharatiya Jyotish Sastra* (1969, p. 345). There Dikshit lists the opinions of 6 authors regarding the identity of the yogataras. For example, for Purvaphalguni the 6 authors put forward 2 different stars as the yogatara of this nakshatra.

These numbers give an indication of the amount of uncertainty that prevails regarding each yogatara. The average numbers of alternatives in the 4 groups are 1.86, 1.83, 2.5, and 1.14, respectively. We can see from this that the greatest degree of uncertainty is found in group 3.

This is the group in which the stars do not tend to move towards the yogatara position as we go back in time. One possible reason for this is that in this group, several of the stars chosen as yogataras are not the actual yogataras of the nakshatras in question, and therefore their reversed proper motions show no tendency to point towards the yogatara positions. The observation that yogatara identifications tend to be more uncertain in this group than in the other groups,

is consistent with this idea (although, of course, it doesn't prove that it is correct).

More evidence supporting this idea is provided in the column under Albiruni. As we pointed out above, in the early 10th century the Muslim astronomer Albiruni tried to ascertain the stars making up the nakshatras. He included a table reporting his findings in his book *Indica* (Sachau 1910, pp. 84–85). Examining this table, we find that either Burgess agrees with Albiruni about the stars in a nakshatra, or else Albiruni lists the nakshatra as unknown. These alternatives, written as "agrees" or "unknown", are listed under Albiruni in Table 4 (p. 35).

In the 4 groups, the numbers of Albiruni's unknown nakshatras are 0, 2, 5, and 1, respectively. This gives additional support to the idea that the identifications in group 3 are less certain than those in the other groups, and that a greater percentage of the yogataras in this group may therefore have been incorrectly chosen.

Burgess (1860) discusses the relationship between the Indian nakshatras and two similar systems of 28 asterisms, one from China and the other from Arabia. Here the term asterism refers to stars or constellations used to mark the passage of the Sun, Moon, and planets as they move along their orbits near the ecliptic.

The Chinese system consists of 28 stars called sieu. In some nakshatras the yogatara selected by Burgess matches the corresponding Chinese sieu; in other nakshatras these do not match, but the sieu is found in the same nakshatra; and in still other cases the sieu is not found in the same nakshatra. These alternatives, written as "yogatara", "in nak.", and "no", are listed in the column under Sieu in Table 4.

In the 4 groups, the numbers of yogataras that match the corresponding sieu are 4, 3, 1, and 0, respectively. Burgess argues that matches between yogataras and sieu are found because there is a historical relationship between the systems of Indian nakshatras and Chinese sieu. If this is correct, then the fact that there are more matches in the first two groups than in the second two tends to support the idea that yogatara identifications are more reliable in the first two groups. These groups (and especially group 1) are the ones in which stars identified as yogataras tend to approach the yogatara positions at dates roughly approximating 50,000 B.P.

The numbers of cases where the sieu does not lie within the nakshatra are 1, 1, 2, and 7, respectively. Here the 4th group is strongly singled out by the fact that none of its nakshatras contain the corresponding sieu. At the same time, the six authors are almost unanimous about these nakshatras, and Albiruni lists only one as unknown.

We do not know the reason for this striking pattern, but we will hazard a guess so as to show that it can be seen as consistent with our overall hypothesis. We note that the stars in group 4 were separated out from the list of 35 stars on the basis of their high D(5)/D ratio. These are stars that tend to move quickly away

from the yogatara position as we move back in time more than a few thousand years. This suggests that these stars may have been chosen as yogataras relatively recently. If the connection with the Chinese system dates to earlier times, this may be why these stars are unrelated to that system.

The Arabs have a traditional system of 28 constellations called manzils, or stations of the Moon. Some nakshatras, as defined by Burgess, are exactly the same as the corresponding Arabian manzils; some are approximately the same; and some show no relationship. These alternatives are listed under Manzil as "exact", "approx.", or "no".

This information concerning manzils reveals the same patterns that we have already observed. Thus, in the first two groups there are 6 exact matches between nakshatras and manzils, but in the second two groups there are only two. Likewise, in the first two groups there are two cases with no relation between nakshatras and corresponding manzils, and in the second two groups there are eight. In general, we see that groups defined on the basis of proper motions of stars show systematic distinctions based on their relationships with the Arabian and Chinese systems of constellations. This is reasonable if there is a relationship between proper motions of stars and the ancient history of all these systems.

4. CONCLUSION

We have offered the tentative hypothesis that the star coordinates found in the *Surya-siddhanta* and other Indian astronomical texts date back to a remote period. Calculations based on proper motions of stars suggest that some of these coordinates may date back to about 50,000 B.C., while others may date back to a few thousand years B.C.

Of course, this is a very radical hypothesis. According to paleoanthropologists, the first positive evidence for the existence of anatomically modern man in Europe or the Middle East dates to about 40,000 years ago at the beginning of the Upper Paleolithic period, and the development of agriculture and village life did not take place until 10,000 to 7,000 years ago (Gowlett, 1984, pp. 120, 156). Thus, from the viewpoint of the paleoanthropologists, it seems very unlikely that people could have measured and recorded positions of stars as far back as 50,000 years ago.

On the other hand, from the point of view of the Vedic and Puranic literatures of India, our hypothesis is not so extraordinary. According to these literatures, human civilization extends back in time for hundreds of thousands or even millions of years. In the Puranas, the period from about 5,000 years ago to 869,000 years ago is known as the Dvapara Yuga. During this period a highly developed civilization is said to have existed, in which knowledge was transmitted by memorization by an organized society of sages.

TABLE 4

Data Concerning the Identification of the 28 Nakshatras, and the Relation of These Nakshatras to the Chinese and Arabian Systems of Asterisms

	No.	Nakshatra	Alt	Albiruni	Sieu	Manzil
Group 1:						
	11	Purvaphalguni	2	agrees	no	exact
	4	Rohini	1	agrees	in nak.	approx.
	3	Krittika	1	agrees	yogatara	exact
	5	Mrigasirsa	2	agrees	yogatara	exact
	13	Hasta	2	agrees	yogatara	no
	19	Mula	4	agrees	in nak.	approx.
	8	Pushya	1	agrees	yogatara	approx.
Group 2:						
	1	Asvini	2	agrees	yogatara	exact
	14	Chitra	1	agrees	yogatara	exact
	25	Satabhisaj	1	unknown	no	no
	18	Jyestha	1	agrees	in nak.	approx.
	2	Bharani	3	may agree	yogatara	exact
	16	Visakha	3	unknown	in nak.	approx.
Group 3:						
	24	Dhanishtha	2	unknown	no	no
	26	Purvabhadrapada	1	unknown	yogatara	approx.
	17	Anuradha	2	agrees	in nak.	exact
	9	Aslesha	4	unknown	in nak.	no
	6	Ardra	3	unknown	no	no
	27	Uttarabhadrapada	2	unknown	in nak.	exact
	20	Purvashadha	2	agrees	in nak.?	approx.
	21	Uttarashadha	4	agrees	in nak.?	no
Group 4:						
	15	Svati	1	agrees	no	no
	7	Punarvasu	1	agrees	no	approx.
	23	Sravana	1	agrees	no	no
	22	Abhijit	1	agrees	no	no
	10	Magha	1	agrees	no	approx.
	28	Revati	2	unknown	no	no
	12	Uttaraphalguni	1	agrees	no	approx.

The descriptions in the Puranas make it clear that this civilization was characterized by great conservatism and resistance to social change. Technical knowledge was restricted to a small class of people; it was transmitted only from

guru to disciple, and was kept secret from the general mass of people on the grounds that it might be abused if it fell into the wrong hands. The use of memorization and the guru-disciple system tended to limit the rate of change of knowledge, and it also tended to preserve old information over long periods.

Even today in India there are brahmins who make a practice of memorizing the Vedas according to the old system. It has been observed that their system of memorization tends to preserve knowledge more effectively than the method of writing books, since books can be changed by careless or unscrupulous copyists (and today such changes can be widely disseminated by means of the printing press). Memorization involves regular recitation before fellow students and teachers, and thus it provides little scope for either mistakes or innovations.

It is far beyond the scope of this article to make a thorough case for the existence of such an ancient civilization, and to reconcile this with the views of modern paleoanthropologists, archeologists, and historians. However, it is interesting to express our hypothesis in terms of the idea that such a civilization may have existed.

If we do this, our hypothesis goes as follows: In the Dvapara Yuga, astronomical knowledge was cultivated, since this knowledge was useful for determining the dates of religious ceremonies and for astrological calculations. The positions of certain important stars were known, and were transmitted by the traditional guru-disciple system. This knowledge was rarely revised; according to the results of our calculations, revisions may have last occurred for many stars about 50,000 years ago.

According to the Puranas, a profound change in human consciousness took place about 5,000 years ago. Here it is not appropriate to discuss in detail the reasons given in the Puranas for this change. However, we can say that it involved a breakdown in the earlier tendency towards social conservatism, and a great increase in the rate of innovation and social fragmentation. The Puranas refer to the period from 5,000 years ago to the present as the Kali Yuga, or the Age of Quarrel.

At the beginning of this period, writing began to replace memorization as a means of transmitting knowledge. According to our hypothesis, this also happened with astronomical knowledge. As the Kali Yuga progressed, a number of changes in star coordinates or identifications were introduced. This accounts for the star coordinates in Table 3 (p. 15) which have high D(5)/D ratios, and which have estimated ages in the order of a few thousand years. The high proper motions of some of these stars may have made it necessary to revise their coordinates at regular intervals.

Gradually, as social fragmentation increased, the old system of transmitting astronomical knowledge began to break down. Civil wars and repeated invasions by foreign conquerors caused many living traditions to be broken, leaving later

generations with the task of trying to reconstruct them from surviving manuscripts.

By about 1,000 years ago, knowledge of the identities of many of the nakshatras was lost. It is significant that this is not book knowledge. The coordinates themselves could be preserved more or less accurately by the process of copying books. But to be able point out stars in the sky one needs a teacher who has learned in turn from an earlier teacher. (This assumes that the books do not have detailed sky maps that a novice could use on his own, and this is the case with texts such as the *Surya-siddhanta*.)

One might ask how the Chinese and Arabic systems of asterisms fit into this hypothesis. As Burgess and others have pointed out, there does seem to be some relationship between these systems of asterisms and the Indian nakshatras, and it stands to reason that this relationship should be due to cultural transmission at some historical period. All we can say about this is that, according to our hypothesis, there are many thousands of years during which the necessary cultural transmissions could have taken place. These transmissions of knowledge could have taken place before the development of many of the ethnic and national divisions which have been so prominent in recent recorded history.

Another question that we must address is this: How did the Indian star coordinates come to be expressed in the particular form in which they are found in the *Surya-siddhanta* and other Indian astronomical texts? As we have noted, these coordinates are expressed in polar longitudes and latitudes, and they agree best with corresponding modern coordinates at an epoch somewhere in the late 5th century A.D. (we used A.D. 490).

There are various possible answers to this question. One possibility is that the star coordinates were transmitted over many thousands of years in the form of ecliptic coordinates, since these do not change as a result of precession of the equinoxes. Of course, this proposal opens the question of why ecliptic coordinates should recently be replaced by polar longitudes and latitudes. Nonetheless, the proposal is plausible, since according to Dikshit (1969, p. 342), ecliptic star coordinates *are* sometimes used in Indian astronomy.

Another possibility is that polar longitudes and latitudes were always used, but they were customarily expressed for an epoch in which Revati, the starting point of the Indian sidereal system, was located at the vernal equinox. Such a system would certainly be convenient, given the rules for astronomical computation presented in the *Surya-siddhanta*.

As it turns out, the existing polar longitudes and latitudes in the *Surya-siddhanta* and other texts are adjusted so that Revati has a longitude of 0 or nearly 0 (see Table 1a, p. 2). Some may argue that this is so because the star coordinates were written down at the particular time when, as a result of the precession of the equinoxes, Revati did lie at the vernal equinox. However, it should be pointed

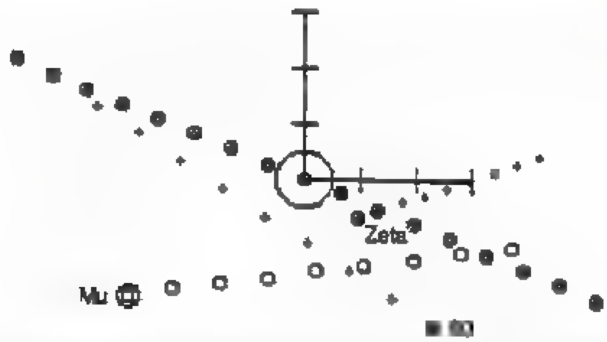


Figure s36. The proper motions of Mu, Zeta, and 80 Piscium over the last 80,000 years.

out that we have no direct historical evidence showing that the Indian star coordinates were first written down at this time.

In fact, this line of reasoning is marred by a serious discrepancy. Colebrooke, Burgess, Pingree, and others say that Revatī corresponds to the star Zeta Piscium. But Zeta Piscium had a longitude of 0 in about A.D. 570. As we can see from Table 3 (p. 15), it was 1.182 degrees away from the position of Revati in A.D. 490, the epoch chosen by Burgess (1860, p. 355) as giving the best overall alignment between modern and Indian star positions. Its distance from Revati's position was even greater in A.D. 425, the date preferred by Pingree (1989, p. 106) for the initial determination of the Indian star coordinates.

What can we say about this discrepancy? As we can see in Figure s34 (p. 30), the proper motion of Zeta Piscium causes it to move away from the position of Revati as we go back in time. In 50,000 B.E. this star would have been 2.989 degrees away from Revati's position. Thus Zeta Piscium is also incompatible with our hypothesis regarding the antiquity of the Indian star coordinates.

But is Zeta Piscium actually the star corresponding to Revati? Albiruni lists the identity of Revati as unknown, and Pingree (1989, p. 106) reluctantly accepts it as Zeta Piscium since "there are no other visible stars in the neighborhood." However, Dikshit (1969, p. 337) suggests that Revati might be Mu Piscium, a star of visual magnitude, 4.84, which is nearly the same as that of Zeta.

In Figure s36 there is a plot showing the motion of Mu Piscium over the last 80,000 years. We can see that Mu Piscium does approach the position of Revati as we go back in time. In A.D. 2000 it is 3.784 degrees from Revati. But it approaches this position at an angle of 26.331 degrees and reaches a minimum distance of 1.677 degrees in 40,307 B.E. It is therefore possible that Mu Piscium may have been used as Revati some 54,000±18,000 years ago, the rough time frame that we computed above for the older star coordinates.

One might ask whether or not any other candidates for the yogatara of Revati might be found by searching through the visible stars in the vicinity of Revati's

position. To test for this possibility, we looked up the coordinates and proper motion parameters for all of the labeled stars within about 10 degrees of Revati on a standard star chart (the *Sky Atlas, Epoch 2000.0*). We found a list of 26 stars.

Of these, there were two which approached Revati at small angles and reached a point of closest approach within the last 80,000 years. One of these was Mu Piscium. The other, which is also shown in Figure s36, is 80 Piscium. This star has a magnitude of 5.52, which is rather low. However, it approaches the position of Revati more closely than Mu Piscium, coming within 0.934 degrees in 37,855 B.E. Its distance from Revati is 3.481 degrees in A.D. 2000, and its angle of approach is 15.571 degrees. Thus 80 Piscium is also a candidate for the yogatara of Revati, although its low magnitude makes it rather unsuitable.

Thus far the dimmest star we have considered as a possible yogatara is Theta Cancri (a candidate for the yogatara of Pushya), which has a magnitude of 5.35. However, Burgess noted that this star was assigned a magnitude of 4 by Ptolemy, which shows that the magnitude of a star can change considerably over several centuries. The second dimmest is Zeta Piscium itself, with a magnitude of 4.86.

We should also make an observation about the procedure of selecting a possible yogatara out the total set of visible stars in a particular region. It can be shown that in the vicinity of Revati, there is about a 20% chance of finding near any given point a visible star that has an angle of approach to that point of less than 30 degrees and a time of closest approach of 54,000±18,000 B.E. (See the appendix for the calculation of this figure.) This means that if we find such a star by searching through the total set of visible stars near Revati, we cannot be highly confident that this is not simply an example of chance alignment.

Therefore, in this paper we have generally restricted ourselves to considering stars that have been mentioned by Burgess and other investigators in their efforts to understand Indian astronomical texts. We offer Mu Piscium, and possibly 80 Piscium, as candidates for the yogatara of Revati only to show that this important nakshatra *may* fit in consistently with our general hypothesis about the antiquity of the Indian star coordinates.

We have two main arguments in favor of this hypothesis. The first is that the restricted set of stars considered by Burgess in 1860 tends to show striking correlations between reversed proper motions and the vectors pointing to corresponding Indian star coordinates, even though Burgess did not consider proper motions when he selected them. The second is that the stars which do not show this correlation show a greater tendency towards uncertainty of identification, as indicated in Table 4 (p. 35), than those which do. This is itself an unexpected correlation between proper motions and historical data (such as Albiruni's list of unknown nakshatras). It also provides a explanation for the stars that do not line up in accordance with proper motions—namely, that some of these may have been selected by mistake.

APPENDIX

Equations Used in the Calculations

We converted the Indian star coordinates from polar longitude and latitude to right ascension and declination by means of the following equations:

$$\begin{aligned} \alpha &= \arctan[\cos(\varepsilon)\sin(\lambda^*),\cos(\lambda^*)] \\ \delta &= \sin^{-1}[\sin(\varepsilon)\sin(\lambda^*)] + \beta^* \end{aligned} \quad (1)$$

Here, λ^* and β^* are the polar longitude and latitude, and α and δ are the right ascension and declination. We use the computer-function arctan(x,y), which automatically gives an arctangent lying in the proper quadrant.

The quantity ε is the obliquity of the ecliptic. Its current value is approximately 23.5 degrees. However, we used 24 degrees, since this is the value used in ancient Indian astronomy, and it is the value that would have been used in India to transform star coordinates *into* polar longitudes and latitudes.

For calculations of precession of the equinoxes, we used the standard equations from Green (1985, p. 219) for the epoch of A.D. 2000.

Points $X=(x,y,z)$ on the celestial sphere can be generated from right ascensions and declinations by the formulas:

$$\begin{aligned} x &= \cos(\delta)\cos(\alpha) \\ y &= \cos(\delta)\sin(\alpha) \\ z &= \sin(\delta) \end{aligned} \quad (2)$$

Given two positions, X and Y, on the celestial sphere, the distance between them, in degrees, is

$$D = \cos^{-1}(X \cdot Y) \quad (3)$$

Proper motion of a star is given rigorously by the formula,

$$X(t) = [(R+tR')X + tR(\delta' X_\delta + \alpha' X_\alpha)]/Z \quad (4)$$

where Z is a normalization constant converting X(t) into a unit vector, and where X_α is:

$$\begin{aligned} x &= -\cos(\delta)\sin(\alpha) \\ y &= \cos(\delta)\cos(\alpha) \\ z &= 0 \end{aligned} \quad (5)$$

and X_s is:

$$x = -\sin(\delta)\cos(\alpha)$$
$$y = -\sin(\delta)\sin(\alpha) \quad (6)$$
$$z = \cos(\delta)$$

The quantities R, R', δ', and α' are radial distance, radial velocity, rate of change in declination, and rate of change in right ascension, all in appropriate units. These quantities are obtained from standard star tables. In practice, R and R' have little effect on the results of our calculations, and thus we can simplify Eqn. (4) by setting R=1 and R'=0. For some stars this is necessary, since R or R' are not known.

For making plots of star positions and proper motions near a position with right ascension, α_0, and declination, δ_0, we used a 2-dimensional Euclidean coordinate system. For the point with position, (α,δ), near (α_0,δ_0), we define x and y in this system by:

$$x = (\alpha_0-\alpha)\cos(\delta)$$
$$y = \delta-\delta_0 \quad (7)$$

Here the α_0-α term gives us a right handed coordinate system, and the $\cos(\delta)$ allows for the fact that the distance represented by a degree of right ascension decreases as we go away from the equator.

From a position X on the celestial sphere, we can compute (α,δ) by inverting Eqn. (2), and then compute (x,y) using Eqn. (7). From X(t) we can similarly compute (x(t),y(t)). It is convenient to define the reversed proper motion vector by

$$(u,v) = (x(2000)-x(-8000),\ y(2000)-y(-8000)), \quad (8)$$

which represents the motion from A.D. 2000 to a time 10,000 years previously. The proper motion is sufficiently linear that we can approximate the position of the star 10,000T years into the past by (x,y)+T(u,v). This expression is written as X+TV in the main body of this article.

We normally define (α_0,δ_0) to be the yogatara position. In that case the error vector giving the difference between the yogatara position and the star position is simply

$$(z,w) = -(x(2000),y(2000)) \quad (9)$$

The angle of approach is then given by

$$A = \cos^{-1}[(uz+vw)/(u^2+v^2)^{.5}(z^2+w^2)^{.5}] \tag{10}$$

The time of closest approach is given by

$$T = (uz+vw)/(u^2+v^2) \tag{11}$$

This T corresponds to a date of 10,000T B.E., where B.E., or before epoch, means before A.D. 2000.

Eqn's. (10) and (11) are approximations that apply to the 2-dimensional coordinate system introduced by Eqn. (7), and they are accurate enough for our application. However, the angle of approach and time of closest approach can also be computed by rigorous formulas based on 3-dimensional vector analysis, and these formulas were used in the calculations. They are:

$$A = \cos^{-1}[V \cdot Y/\sin(D)] \tag{12}$$

where D is as in Eqn. (3), V is

$$V = -[\delta' X_\delta + \alpha' X_\alpha]/Z \tag{13}$$

and Z is a normalization constant making V a unit vector. Likewise, T is given by the rigorous formula,

$$T = [(X' \cdot Y)(X \cdot X)-(X \cdot Y)(X' \cdot X)]/[(X \cdot Y)(X' \cdot X')-(X' \cdot Y)(X' \cdot X)] \tag{14}$$

where X' is

$$X' = -[R'X + R(\delta' X_\delta + \alpha' X_\alpha)] \tag{15}$$

In (13) and (15) the minus sign is there to give us a reversed proper motion vector.

As a final point, let us estimate the chance of finding near any given point a visible star that (1) has an angle of approach to that point of less than 30 degrees and (2) has a time of closest approach of 54,000±18,000 B.E. We will estimate this for a region surrounding the position of Revati that extends from 0 to 16 degrees of declination, and spans 25 degrees in right ascension.

It can be shown that for a star with proper motion of v arc seconds per year, the area of the sky containing points satisfying conditions (1) and (2) is approximately

$$\text{Area} = 4TS(v^2)\tan(A)/3600^2 \tag{16}$$

where T=54,000 years, S=18,000 years, A=30 degrees, and the 3600^2 converts Area into square degrees. If we add up these areas for the 26 visible stars that we identified in the 16×25 degree region, we get a total of 72.14 square degrees. Of course, this is an upper bound on the total area, because some of the 26 areas will overlap. The area of the 16×25 degree region is 394.82 square degrees. Therefore, the area satisfying conditions (1) and (2) is somewhat less than 18.3% of the area of the region as a whole. This percentage is an upper bound on the probability that a randomly chosen point in the region will satisfy conditions (1) and (2) for one of the region's visible stars. (Since the density of the visible stars varies, this probability will be somewhat different in other regions of the sky.)

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